Day 25 **S Tier 10-a-day: Model Solutions**

1 Percentages

B is 10% more than D.
D is 5% more than P.
Work out B as a percentage of P.

B = 1.1 × D and D = 1.05 × P
B = 1.1 × 1.05 × P = 1.155 × P
1.155 × 100 = 115.5%
Answer: 115.5%

2 Mean of a changing set

The mean of 7 numbers is 153.
One number is removed and the mean of the remaining 6 numbers is 174.
Work out the number that was removed.

Total of the 7 numbers = 7 × 153 = 1071
Total of the 6 remaining = 6 × 174 = 1044
Number removed = 1071 − 1044 = 27
Answer: 27

3 Equations with fractions

Solve $\frac{x+1}{2} + \frac{x-7}{6} = 2$

Multiply every term by 6: 3(x + 1) + (x − 7) = 12
Expand: 4x − 4 = 12
4x = 16
x = 4
Answer: x = 4

4 Using a line of best fit

The scatter graph shows distance from town (x) and price (y).
The equation of the line of best fit is y = −1.5x + 23.
(a) Estimate y when x = 7.
(b) Explain why an estimate of y when x = 20 may be unreliable.

(a) y = −1.5 × 7 + 23 = 12.5
(b) x = 20 is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)
Answer: (a) 12.5 (b) x = 20 is outside the range of the data

5 Column vectors

$\mathbf{a} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$
$\mathbf{a} + k\mathbf{b}$ is parallel to $\mathbf{c}$.
Work out the value of k.

$\mathbf{a} + k\mathbf{b} = \begin{pmatrix} 3 - 1k \\ -4 + 1k \end{pmatrix}$
Parallel to $\mathbf{c}$: $\frac{3 - 1k}{2} = \frac{-4 + 1k}{1}$
1(3 − 1k) = 2(−4 + 1k)
k × (−3) = −11
$k = \frac{11}{3}$
Answer: $\mathbf{k = \frac{11}{3}}$

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6 Simultaneous equations

Solve the simultaneous equations
$6x + 4y = -38$
$2x - 5y = 19$

To match the x terms: (2) × 3 gives $6x - 15y = 57$
Subtract the equations to eliminate x: $19y = -95$
$y = -5$
Substitute $y = -5$ into (1): $6x - 20 = -38$
$6x = -18$, so $x = -3$
Answer: $x = -3$, $y = -5$

7 Percentages of percentages

In a group of employees, 55% are men and the rest are women.
40% of the men and 25% of the women work from home.
What percentage of all the employees work from home?

Take 100 employees: 55 are men and 45 are women
40% of 55 = 22 and 25% of 45 = 11.25
22 + 11.25 = 33.25 out of 100
Answer: 33.25%

8 HCF and LCM

(a) Write 252 and 1008 as products of prime factors.
(b) Find the HCF and the LCM of 252 and 1008.
Give your answers as numbers.

(a) $252 = 2^2 \times 3^2 \times 7$ and $1008 = 2^4 \times 3^2 \times 7$
(b) HCF = $2^2 \times 3^2 \times 7 = 252$
LCM = $2^4 \times 3^2 \times 7 = 1008$
Answer: (a) $2^2 \times 3^2 \times 7$; $2^4 \times 3^2 \times 7$ (b) HCF = 252, LCM = 1008

9 Tree diagrams without replacement

A bag contains 5 yellow beads and 4 white beads.
Two beads are taken at random without replacement.
Work out the probability that (a) both are the same colour, (b) exactly one is white.

(a) Add the ways: $\frac{5}{9}\times\frac{4}{8} + \frac{4}{9}\times\frac{3}{8} = \frac{4}{9}$
(b) P(exactly one white) = $\frac{4}{9}\times\frac{5}{8} + \frac{5}{9}\times\frac{4}{8} = \frac{5}{9}$
Answer: (a) $\frac{4}{9}$ (b) $\frac{5}{9}$

10 Cosine rule and area of a triangle

The diagram shows triangle ABC. (Calculator)
(a) Work out the length of AB, marked x, to 1 decimal place.
(b) Work out the area of triangle ABC to 3 significant figures.
The diagram is not drawn accurately.

(a) $x^2 = 7^2 + 5^2 - 2 \times 7 \times 5 \times \cos 70°$
$x^2 = 50.059$
x = 7.1 cm (1 d.p.)
(b) Area = $\frac{1}{2} \times 7 \times 5 \times \sin 70° = 16.4$ cm^2 (3 s.f.)
Answer: (a) 7.1 cm (b) 16.4 cm^2