Day 249 **S Tier 10-a-day: Model Solutions**

1 Area of a triangle using sine

Triangle ABC has AB = 20 cm and BC = 15 cm.
The area of triangle ABC is $75\sqrt{3}$ cm^2.
Angle ABC is obtuse.
Work out the size of angle ABC.
Do not use a calculator.

Area = $\frac{1}{2} \times AB \times BC \times \sin B$

$75\sqrt{3} = \frac{1}{2} \times 20 \times 15 \times \sin B$

$\sin B = 75\sqrt{3} \div 150 = \frac{\sqrt{3}}{2}$

$\sin 60° = \frac{\sqrt{3}}{2}$, and B is obtuse, so $B = 180° - 60° = 120°$

Answer: 120°

2 Equations with fractions

Solve $\frac{x-1}{3} + \frac{x+1}{2} = -4$

Multiply every term by 6: $2(x - 1) + 3(x + 1) = -24$
Expand: $5x + 1 = -24$
$5x = -25$
$x = -5$
Answer: x = −5

3 Exact trigonometric values

Work out the exact value of sin 60° × sin 45°
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$\sin 60° \times \sin 45° = \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}}{4}$

Answer: $\frac{\sqrt{6}}{4}$

4 Comparing costs

FitLife has a joining fee of £2.50 plus £4.50 per month.
GymNow has a joining fee of £9.50 plus £4 per month.
(a) Work out the number of months for which the two costs are the same.
(b) Which is cheaper for 19 months, and by how much?

(a) Let n be the number of months. Working in pounds, the costs are equal when $2.50 + 4.50n = 9.50 + 4n$
$0.50n = 7$, so $n = 14$
(b) FitLife: £2.50 + £4.50 × 19 = £88
GymNow: £9.50 + £4 × 19 = £85.50
GymNow is cheaper by £2.50
Answer: (a) 14 (b) GymNow, by £2.50

5 Distance-time graphs

The graph shows Priya's cycle ride to a café and back.
(a) Work out Priya's speed on the way back.
(b) Work out Priya's average speed for the whole ride, including the stop.

(a) Return: 24 km in 1 hours, so speed = 24 ÷ 1 = 24 km/h
(b) Total distance = 2 × 24 = 48 km, total time = 4 hours
Average speed = 48 ÷ 4 = 12 km/h
Answer: (a) 24 km/h (b) 12 km/h

Day 249 **S Tier 10-a-day: Model Solutions**

6 Income tax

In one country, income tax is worked out like this:
The first £12,570 of income is not taxed.
The next £20,000 is taxed at 20%.
All income above that is taxed at 45%.
Lena pays £14,125 in income tax in a year.
Work out Lena's income for the year.

Tax on the 20% band = 20% of £20,000 = £4,000
Tax paid at 45% = £14,125 − £4,000 = £10,125
Income taxed at 45% = £10,125 ÷ 0.45 = £22,500
Income = £12,570 + £20,000 + £22,500 = £55,070
Answer: £55,070

7 Direct proportion formulae

y is directly proportional to the cube of x.
When x = 3, y = 54.
(a) Find a formula for y in terms of x.
(b) Work out the value of y when x = 5.

(a) $y = kx^3$
$54 = k \times 27$, so $k = 54 \div 27 = 2$
$y = 2x^3$
(b) $y = 2 \times 125 = 250$
Answer: (a) $y = 2x^3$ (b) 250

8 Exact lengths and areas

A rectangle has area $(2\sqrt{5})$ cm^2 and length $(4 + 2\sqrt{5})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{5}$

Width = area ÷ length = $\frac{2\sqrt{5}}{4 + 2\sqrt{5}}$
Multiply top and bottom by $(4 - 2\sqrt{5})$:
Numerator: $(2\sqrt{5})(4 - 2\sqrt{5}) = -20 + 8\sqrt{5}$
Denominator: $16 - 20 = -4$
Width = $5 - 2\sqrt{5}$
Answer: $5 - 2\sqrt{5}$ cm

9 Tangent to a circle

A circle has equation $(x - 2)^2 + (y - 4)^2 = 100$.
The point P(10, 10) lies on the circle.
Find an equation of the tangent to the circle at P.
Give your answer in the form ax + by = c, where a, b and c are integers.

The centre is (2, 4). The gradient of the radius to P is $\frac{10 - 4}{10 - 2} = \frac{3}{4}$
The tangent is perpendicular to the radius, so its gradient is $-\frac{4}{3}$. Using P: $y - 10 = -\frac{4}{3}(x - 10)$
Multiply by 3: $3(y - 10) = -4(x - 10)$
Expand and rearrange: $4x + 3y = 70$
Answer: $4x + 3y = 70$

Day 249 **S Tier 10-a-day: Model Solutions**

10 Tests of divisibility

The 4-digit number 16k0 is a multiple of 6.
k is a single digit.
Find all the possible values of k.

A multiple of 6 is even and a multiple of 3
The last digit is 0, which is even
Digit sum = 1 + 6 + 0 + k = 7 + k
This must be a multiple of 3, so k = 2 or 5 or 8
Answer: k = 2 or 5 or 8