Day 248 **S Tier 10-a-day: Model Solutions**

1 Equations with fractions

Solve $\frac{x+6}{5} + \frac{x-4}{2} = -5$

Multiply every term by 10: $2(x + 6) + 5(x - 4) = -50$
Expand: $7x - 8 = -50$
$7x = -42$
$x = -6$
Answer: $x = -6$

2 Error intervals

The volume of juice is 0.19 litres, truncated to 2 decimal places.
Write down the error interval for the volume of juice, using v.

Truncating cuts off the later digits without rounding.
The smallest possible value is 0.19 and the value must be less than 0.2
$0.19 \leq v < 0.2$
Answer: $0.19 \leq v < 0.2$

3 Mean of a changing set

The mean of 8 numbers is 119.
A new number is added and the mean of all 9 numbers is 108.
Work out the new number.

Total of the 8 numbers = $8 \times 119 = 952$
New total = $9 \times 108 = 972$
New number = $972 - 952 = 20$
Answer: 20

4 HCF and LCM

$A = 3 \times 5^3$
$B = 2^2 \times 3^4 \times 5$
Write down, as a product of prime factors,
(a) the HCF of A and B
(b) the LCM of A and B

HCF: take the lowest power of each prime that appears in both numbers
$= 3 \times 5$
LCM: take the highest power of every prime that appears
$= 2^2 \times 3^4 \times 5^3$
Answer: (a) 3×5 (b) $2^2 \times 3^4 \times 5^3$

5 Gradient of a chord

A and B are points on the curve $y = x^2 + x - 4$.
The x coordinate of A is 0 and the x coordinate of B is 3.
Work out the gradient of the line AB.

At A: $y = -4$. At B: $y = 8$
A is $(0, -4)$ and B is $(3, 8)$
Gradient = $\frac{8-(-4)}{3-0} = \frac{12}{3} = 4$
Answer: 4

6 Counting factors

$N = 2^4 \times 3^3$
How many of the factors of N are odd numbers?

An odd factor contains no 2s
Ignore the 2 and count the factors of the rest: 4 = 4
Answer: 4

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7 Pythagoras with algebra

ABC is a right-angled triangle with sides (x + 6) cm, (2x + 2) cm and (x + 8) cm. The longest side is (x + 8) cm.
(a) Find the value of x.
(b) Work out the area of the triangle.
The diagram is not drawn accurately.

(a) Pythagoras: $(x + 6)^2 + (2x + 2)^2 = (x + 8)^2$
Expand and collect terms: $4x^2 + 4x - 24 = 0$
Factorise: $4(x - 2)(x + 3) = 0$
$x = 2$ or $x = -3$, and x must be positive, so $x = 2$
(b) The two shorter sides are 8 cm and 6 cm
Area = $\frac{1}{2} \times 8 \times 6 = 24$ cm^2

Answer: (a) x = 2 (b) 24 cm^2

8 Angles of elevation

A point A is on level ground in line with the base of a lighthouse. (Calculator)
The angle of elevation of the top of the lighthouse from A is 58°.
B is a point on the same line, 60 m further from the lighthouse than A.
The angle of elevation of the top of the lighthouse from B is 20°.
Work out the height of the lighthouse. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 58° = \frac{h}{x}$, so $x = h \div \tan 58°$

From B: $\tan 20° = \frac{h}{x + 60}$, so $x + 60 = h \div \tan 20°$
Subtract: $60 = h \div \tan 20° - h \div \tan 58° = h(2.7475 - 0.62487)$
$h = 60 \div 2.1226 = 28.3$ m (1 d.p.)
Answer: 28.3 m

9 Mixtures and percentage profit

A shop mixes two types of mixed nuts.
Type A costs £10.00 per kg and type B costs £6.00 per kg.
They are mixed in the ratio A : B = 3 : 5.
The mixture is sold for £13.50 per kg.
Work out the percentage profit.

Cost of 8 kg of mixture = 3 × £10.00 + 5 × £6.00 = £60.00
Cost per kg = £60.00 ÷ 8 = £7.50
Profit per kg = £13.50 − £7.50 = £6.00
Percentage profit = 6.00 ÷ 7.50 × 100 = 80%
Answer: 80%

10 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $4\mathbf{b}$, OQ = $5\mathbf{a} + 7\mathbf{b}$ and OR = $25\mathbf{a} + 19\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $5\mathbf{a} + 3\mathbf{b}$
QR = OR − OQ = $20\mathbf{a} + 12\mathbf{b}$
QR = 4 PQ, so PQ and QR are parallel, and they share the point Q
So P, Q and R lie on a straight line
(b) PQ : QR = 1 : 4
Answer: (b) 1 : 4