Day 247 **S Tier 10-a-day: Model Solutions**

1 Multiplying and dividing surds

Simplify fully $\frac{\sqrt{45}}{3\sqrt{5}}$

$\sqrt{45} = 3\sqrt{5}$

$\frac{\sqrt{45}}{3\sqrt{5}} = \frac{3\sqrt{5}}{3\sqrt{5}}$

$= 1$

Answer: 1

2 Mean of a changing set

The mean of 8 numbers is 178.
One number is removed and the mean of the remaining 7 numbers is 196.
Work out the number that was removed.

Total of the 8 numbers = $8 \times 178 = 1424$
Total of the 7 remaining = $7 \times 196 = 1372$
Number removed = $1424 - 1372 = 52$
Answer: 52

3 Percentage change

The population of a village decreased by 10% in 2023.
In 2024 the population decreased by a further 60%.
Work out the overall percentage decrease over the two years.

Multipliers: 0.9 and 0.4
Overall multiplier = $0.9 \times 0.4 = 0.36$
$0.36 - 1 = -0.64$, so an overall decrease of 64%
Answer: 64% decrease

4 Where a line meets the axes

The line L has equation $24x + 7y = 168$.
L crosses the x-axis at the point A and the y-axis at the point B.
(a) Find the coordinates of A and B.
(b) Work out the area of triangle OAB, where O is the origin.

(a) At A, $y = 0$: $24x = 168$, so $x = 7$. A is (7, 0)
At B, $x = 0$: $7y = 168$, so $y = 24$. B is (0, 24)
(b) OA = 7 and OB = 24, and angle AOB = 90°

Area = $\frac{1}{2} \times 7 \times 24 = 84$

Answer: (a) A(7, 0), B(0, 24) (b) 84

5 Enlargement

Triangle T has vertices A(0, −5), B(−1, −5), C(0, 0).
T is enlarged with scale factor −2 and centre (−1, −2).
(a) Write down the coordinates of the image of A.
(b) The area of T is 8 square units. Work out the area of the image of T.

(a) Vector from the centre to A = $(0 - (-1), -5 - (-2)) = (1, -3)$
Multiply by −2: (−2, 6)
Image of A = centre + this vector = (−3, 4)
(b) Area scale factor = $(-2)^2 = 4$
Area = $8 \times 4 = 32$ square units
Answer: (a) (−3, 4) (b) 32

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6 Equations with indices

Solve $4^{x-3} = 16^{x+1}$

Write both sides as powers of 2: $(2^2)^{x-3} = (2^4)^{x+1}$

$2^{2x-6} = 2^{4x+4}$

Equate the powers: $2x - 6 = 4x + 4$

$-2x = 10$

$x = -5$

Answer: $x = -5$

7 Shaded areas: exact answers

The diagram shows a square inside a circle of radius 5 cm.
The four vertices of the square lie on the circle.
Work out the area of the shaded region. Give your answer in terms of π.

The diagonal of the square is a diameter of the circle: 10 cm

Area of the square = $\frac{1}{2} \times$ diagonal $\times$ diagonal = $\frac{1}{2} \times 10 \times 10 = 50$

Area of circle = $\pi \times 5^2 = 25\pi$

Shaded area = $25\pi - 50$ cm^2

Answer: $(25\pi - 50)$ cm^2

8 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve with a turning point at $(3, -4)$.
The curve crosses the y-axis at $(0, 5)$.
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The turning point is $(3, -4)$, so $y = a(x - 3)^2 - 4$

When $x = 0$, $y = 5$: $a \times 9 - 4 = 5$, so $a = 1$

$y = (x - 3)^2 - 4$

Expand: $y = x^2 - 6x + 5$

Answer: $y = x^2 - 6x + 5$

9 Nested surds

Show that $\sqrt{72 + 18\sqrt{7}} = 3 + 3\sqrt{7}$

Square the right-hand side: $(3 + 3\sqrt{7})^2 = 9 + 18\sqrt{7} + 63$

$= 72 + 18\sqrt{7}$

So $\sqrt{72 + 18\sqrt{7}} = 3 + 3\sqrt{7}$

Answer: $\sqrt{72 + 18\sqrt{7}} = 3 + 3\sqrt{7}$

10 Inverse proportion

y is inversely proportional to the square root of x.
When $x = 9$, $y = 15$.
Find the value of y when $x = 25$.

$y = \frac{k}{\sqrt{x}}$

$15 = \frac{k}{3}$, so $k = 15 \times 3 = 45$

$y = \frac{45}{5} = 9$

Answer: $y = 9$