Day 246 **S Tier 10-a-day: Model Solutions**

1 Completing the square

(a) Write $3x^2 + 30x + 84$ in the form $a(x + p)^2 + q$

(b) Write down the coordinates of the turning point of the curve $y = 3x^2 + 30x + 84$

(a) Take out the factor 3 from the x terms: $3(x^2 + 10x) + 84$

$x^2 + 10x = (x + 5)^2 - 25$, so $3x^2 + 30x + 84 = 3[(x + 5)^2 - 25] + 84$

$= 3(x + 5)^2 + 9$

(b) The minimum is when $x + 5 = 0$, so $x = -5$ and $y = 9$

Answer: (a) $3(x + 5)^2 + 9$ (b) $(-5, 9)$

2 Radius from chord and height of arc

A circular arc has a chord AB of length 28 cm.

The greatest distance between the chord and the arc is 2 cm.

Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so OM = r − 2

AM = 28 ÷ 2 = 14. Pythagoras in triangle OMA: $r^2 = (r - 2)^2 + 14^2$

$r^2 = r^2 - 4r + 4 + 196$

4r = 200, so r = 50 cm

Answer: 50 cm

3 Time zones

A flight leaves London at 13:55 (London time).

It takes 7 hours 50 minutes.

When it is 12:00 in London it is 14:00 in Cairo.

At what time, in Cairo time, does the flight land? Say if it is on a different day.

Arrival in London time = 13:55 + 7 h 50 min = 21:45

Cairo is 2 hours ahead of London, so add 2 hours

23:45

Answer: 23:45 the same day

4 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle ACB = 67°.

(a) Work out the size of angle AOB.

(b) Work out the size of angle APB.

The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: 2 × 67 = 134°

(b) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)

Angle APB = 360 − 90 − 90 − 134 = 46°

Answer: (a) 134° (b) 46°

5 Percentages

C is 30% less than S.

S is 60% less than D.

Work out C as a percentage of D.

C = 0.7 × S and S = 0.4 × D

C = 0.7 × 0.4 × D = 0.28 × D

0.28 × 100 = 28%

Answer: 28%

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6 Estimating the median from grouped data

The table gives information about 118 masses.
Work out an estimate for the median.
Assume the values in each class are evenly spread.

Cumulative frequencies: 30, 50, 65, 82, 101, 118
The median is the value at position 118 ÷ 2 = 59
It is in the class 40 to 60, with 50 values before it, so we need 9 more of the 15 values in this class

Median $\approx 40 + \frac{9}{15} \times 20 = 52$

Answer: 52

7 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 15^2 + 6^2 = 261$
$AG^2 = AC^2 + CG^2 = 261 + 10^2 = 361$
AG = 19 cm

(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{10}{\sqrt{261}}$

angle GAC = 31.8° (1 d.p.)
Answer: (a) 19 cm (b) 31.8°

8 Changing the subject

Make x the subject of $y = 2\sqrt{x + 1} - 5$

Add 5: $y + 5 = 2\sqrt{x + 1}$

Divide by 2: $\frac{y + 5}{2} = \sqrt{x + 1}$

Square both sides: $\frac{(y + 5)^2}{4} = x + 1$

$x = \frac{(y + 5)^2}{4} - 1$

Answer: $x = \frac{(y + 5)^2}{4} - 1$

9 Counting factors

$N = 2^2 \times 3^2 \times 7$
How many of the factors of N are even numbers?

An even factor must contain at least one 2
Power of 2: 1 to 2, that is 2 choices
Power of 3: 0 to 2, that is 3 choices
Power of 7: 0 to 1, that is 2 choices
Total = 2 × 3 × 2 = 12
Answer: 12

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10 Probability from a frequency tree

In a population of 10,000 people, 2% have a certain disease.
A test gives a positive result for 98% of people who have the disease and for 3% of people who do not.
A person is chosen at random and tests positive.
Work out the probability that they have the disease.

2% of 10,000 = 200 have the disease and 9,800 do not
Flagged and genuine: 200 × 98% = 196
Flagged but not: 9,800 × 3% = 294
Total flagged or positive = 196 + 294 = 490

Probability = 196 ÷ 490 = $\frac{2}{5}$

Answer: $\frac{2}{5}$