**Day 245** **S Tier 10-a-day: Model Solutions**

## 1 Radius from chord and height of arc

A circular arc has a chord AB of length 16 cm.
The greatest distance between the chord and the arc is 2 cm.
Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so OM = $r - 2$

AM = $16 \div 2 = 8$. Pythagoras in triangle OMA: $r^2 = (r - 2)^2 + 8^2$

$r^2 = r^2 - 4r + 4 + 64$

$4r = 68$, so $r = 17$ cm

**Answer: 17 cm**

## 2 Speed-time graphs

The graph shows the speed of a car during 24 seconds.
(a) Work out the acceleration during the first 6 seconds.
(b) Work out the average speed over the whole 24 seconds.

(a) Acceleration = change in speed ÷ time = $(12 - 0) \div 6 = 2$ m/s$^2$

(b) Distance = area under the graph

Trapezium: $\frac{0 + 12}{2} \times 6 = 36$; rectangle: $12 \times 12 = 144$; triangle: $\frac{1}{2} \times 6 \times 12 = 36$

Total = 216 m, so average speed = $216 \div 24 = 9$ m/s

**Answer: (a) 2 m/s$^2$ (b) 9 m/s**

## 3 Quadratic inequalities

Solve $2x^2 + 2x - 40 < 0$

Divide by 2: $x^2 + x - 20 < 0$

Solve $x^2 + x - 20 = 0$: $(x + 5)(x - 4) = 0$, so $x = -5$ or $x = 4$

The graph is a U shape crossing the x-axis at $-5$ and 4; it is below the axis between the roots

$-5 < x < 4$

**Answer: $-5 < x < 4$**

## 4 Simplifying surds

Write $\sqrt{50} + \sqrt{18} + \sqrt{72}$ in the form $a\sqrt{2}$
where a is an integer.

$\sqrt{50} = 5\sqrt{2}$; $\sqrt{18} = 3\sqrt{2}$; $\sqrt{72} = 6\sqrt{2}$

$(5 + 3 + 6)\sqrt{2} = 14\sqrt{2}$

**Answer: $14\sqrt{2}$**

## 5 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the length of the chord AB.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AM^2 = OA^2 - OM^2 = 26^2 - 24^2 = 100$

AM = 10 cm

AB = $2 \times 10 = 20$ cm

(b) sin(angle AOM) = $\frac{10}{26}$

Angle AOM = 22.62°, so angle AOB = $2 \times 22.62 = 45.2°$

**Answer: (a) 20 cm (b) 45.2°**

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## 6 Line and curve intersections

Solve the simultaneous equations

$y = x^2 - 4x - 1$

$y = 2x - 6$

Substitute: $x^2 - 4x - 1 = 2x - 6$

Rearrange: $x^2 - 6x + 5 = 0$

Factorise: $(x - 1)(x - 5) = 0$, so $x = 1$ or $x = 5$

When $x = 1$: $y = 2 \times 1 - 6 = -4$

When $x = 5$: $y = 2 \times 5 - 6 = 4$

**Answer: $x = 1$, $y = -4$ and $x = 5$, $y = 4$**

## 7 Sample space and conditional probability

A spinner numbered 1 to 4 and a spinner numbered 1 to 5 are each thrown once. The first score and the second score are recorded.

(a) Work out the probability that the sum is a multiple of 3.

(b) Given that the first number is greater than the second, work out the probability that the sum is a multiple of 3.

(a) There are $4 \times 5 = 20$ equally likely outcomes, and 7 of them satisfy: the sum is a multiple of 3

Probability = $\frac{7}{20}$

(b) 6 outcomes satisfy: the first number is greater than the second. Of these, 2 also satisfy: the sum is a multiple of 3

Probability = $\frac{1}{3}$

**Answer: (a) $\frac{7}{20}$ (b) $\frac{1}{3}$**

## 8 Laws of indices

Write $\frac{16^{n+2} \times 2^{n-3}}{4^n}$ in the form $2^{an+c}$

where a and c are integers.

$16 = 2^4$, $2 = 2^1$, $4 = 2^2$

$16^{n+2} = 2^{4(n+2)}$, $2^{n-3} = 2^{n-3}$, $4^n = 2^{2n}$

Exponent: $4(n + 2) + (n - 3) - 2n = 3n + 5$

**Answer: $a = 3$, $c = 5$**

## 9 Composite functions: fg and gf

$f(x) = -x - 3$ and $g(x) = x^2 - 6$

Solve $fg(x) = gf(x)$

$fg(x) = f(x^2 - 6) = -x^2 + 3$

$gf(x) = g(-x - 3) = (-x - 3)^2 - 6 = x^2 + 6x + 3$

Set them equal and rearrange: $2x^2 + 6x = 0$

Divide by 2 and factorise: $x(x + 3) = 0$

$x = -3$ or $x = 0$

**Answer: $x = -3$ or $x = 0$**

## 10 Estimating the mean from a histogram table

The table gives information about the times of some people.

Work out an estimate for the mean time.

Give your answer correct to 1 decimal place. (Calculator)

Frequency = frequency density × class width: 5, 9, 15, 14

Mid-points: 2.5, 7.5, 15, 30

$\Sigma fm = 5 \times 2.5 + 9 \times 7.5 + 15 \times 15 + 14 \times 30 = 725$, $\Sigma f = 43$

Mean = $725 \div 43 = 16.9$

**Answer: 16.9**