Day 244 **S Tier 10-a-day: Model Solutions**

1 Inverse proportion in context

15 machines can make a batch of bolts in 20 hours.
After 2 hours, 5 of the machines leave. The rest carry on working at the same rate.
How many more hours does it take to finish?

Total work = 15 × 20 = 300 machine-hours
Work done = 15 × 2 = 30, so work left = 270
Workers left = 15 − 5 = 10
Time = 270 ÷ 10 = 27 hours
Answer: 27 hours

2 Arc length and sector area

A sector of a circle has radius 5 cm and angle 270°.
(a) Work out the length of the arc of the sector.
(b) Work out the area of the sector. Give each answer in terms of π.

(a) Fraction of the circle = $\frac{270}{360}$

Arc length = $\frac{270}{360} \times 2 \times \pi \times 5 = \frac{15}{2}\pi$ cm

(b) Area = $\frac{270}{360} \times \pi \times 5^2 = \frac{75}{4}\pi$ cm^2

Answer: (a) $\frac{15}{2}\pi$ cm (b) $\frac{75}{4}\pi$ cm^2

3 Using a line of best fit

The scatter graph shows distance from town (x) and price (y).
The equation of the line of best fit is y = −3x + 38.
(a) Estimate y when x = 7.
(b) Explain why an estimate of y when x = 15 may be unreliable.

(a) y = −3 × 7 + 38 = 17
(b) x = 15 is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)
Answer: (a) 17 (b) x = 15 is outside the range of the data

4 Completing the square

(a) Write $x^2 - 2x - 3$ in the form $(x + p)^2 + q$
(b) Write down the coordinates of the turning point of the curve $y = x^2 - 2x - 3$

(a) Half of −2 is −1, so $x^2 - 2x = (x - 1)^2 - 1$
$x^2 - 2x - 3 = (x - 1)^2 - 1 - 3$
$= (x - 1)^2 - 4$
(b) The minimum is when $x - 1 = 0$, so x = 1 and y = −4
Answer: (a) $(x - 1)^2 - 4$ (b) (1, −4)

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5 Vectors in a triangle

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 1. N is on AB with AN : NB = 1 : 1.
(a) Find ON in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Show that MN is parallel to OB.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{1}{2}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

(b) OM = $\frac{1}{2}\mathbf{a}$

MN = ON − OM = $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a} = \frac{1}{2}\mathbf{b}$

MN = $\frac{1}{2}\mathbf{b}$, which is a multiple of OB, so MN is parallel to OB

Answer: (a) $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$ (b) MN = $\frac{1}{2}\mathbf{b}$, parallel to OB

6 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 10 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 20 ÷ 10 = 2 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 10 \times 20 + 20 \times 15 + \frac{1}{2} \times 10 \times 20 = 100 + 300 + 100$
= 500
Answer: (a) 2 m/s^2 (b) 500 m

7 Recasting solids

A solid cylinder of radius 6 cm and height 11 cm is melted down.
The metal is used to make identical solid spheres of radius 3 cm.
Work out the number of spheres that can be made.

The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

Volume of cylinder = $\pi \times 6^2 \times 11 = 396\pi$

Volume of one sphere = $\frac{4}{3} \times \pi \times 3^3 = 36\pi$

Number of spheres = 396 ÷ 36 = 11
Answer: 11

8 Surds with unknowns

Given that $(a + \sqrt{7})^2 = 88 + 18\sqrt{7}$,
where a is a positive integer, find the value of a.

$(a + \sqrt{7})^2 = a^2 + 2a\sqrt{7} + 7$
Compare the $\sqrt{7}$ terms: 2a = 18, so a = 9
Check the integer terms: $a^2 + 7 = 81 + 7 = 88$
Answer: a = 9

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9 Functions that are their own inverse

$f(x) = \frac{3x - 1}{x - 3}$ where $x \neq 3$

(a) Show that $ff(x) = x$.

(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by $(x - 3)$

$ff(x) = \frac{3(3x - 1) + (-1)(x - 3)}{(3x - 1) - 3(x - 3)}$

Top $= 9x - 3 - x + 3 = 8x$

Bottom $= 3x - 1 - 3x + 9 = 8$

$ff(x) = \frac{8x}{8} = x$

(b) $ff(x) = x$, so f is its own inverse: $f^{-1}(x) = \frac{3x - 1}{x - 3}$

Answer: (b) $\mathbf{f^{-1}(x) = \frac{3x - 1}{x - 3}}$

10 Nested surds

Show that $\sqrt{10 + 4\sqrt{6}} = 2 + \sqrt{6}$

Square the right-hand side: $(2 + \sqrt{6})^2 = 4 + 4\sqrt{6} + 6$

$= 10 + 4\sqrt{6}$

So $\sqrt{10 + 4\sqrt{6}} = 2 + \sqrt{6}$

Answer: $\mathbf{\sqrt{10 + 4\sqrt{6}} = 2 + \sqrt{6}}$