Day 243 **S Tier 10-a-day: Model Solutions**

1 Exact trigonometric ratios from a given ratio

x is an acute angle and $\cos x = \frac{8}{13}$.

(a) Find the exact value of sin x.

(b) Find the exact value of tan x. Give each answer as a fraction or a surd. Do not use a calculator.

(a) $\sin^2 x + \cos^2 x = 1$, so $\sin^2 x = 1 - (\frac{8}{13})^2 = \frac{105}{169}$

x is acute, so sin x is positive: $\sin x = \frac{\sqrt{105}}{13}$

(b) $\tan x = \frac{\sin x}{\cos x} = \frac{\sqrt{105}}{13} \div \frac{8}{13} = \frac{\sqrt{105}}{8}$

Answer: (a) $\frac{\sqrt{105}}{13}$ (b) $\frac{\sqrt{105}}{8}$

2 Factorising quadratics

Factorise $2x^2 - 11x - 21$

$2 \times (-21) = -42$. Two numbers that multiply to -42 and add to -11 are 3 and -14

Split the middle term: $2x^2 + 3x - 14x - 21$

Factorise in pairs: $x(2x + 3) - 7(2x + 3)$

$= (x - 7)(2x + 3)$

Answer: $(x - 7)(2x + 3)$

3 Exponential growth and decay

A population of members of a website starts at 200 and grows by 50% each year.

After how many whole years will the population first be at least double its starting value? (Calculator)

Multiplier = 1.5, double is 400

After 1 years: $200 \times 1.5^1 = 300.0$

After 2 years: $200 \times 1.5^2 = 450.0$

Answer: 2 years

4 Exact trigonometric values

Work out the exact value of $\tan 30° \div \sin 45°$

Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$\tan 30° \div \sin 45° = \frac{\sqrt{3}}{3} \div \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}}{3}$

Answer: $\frac{\sqrt{6}}{3}$

5 Fractional and negative indices

Work out $16^{-\frac{7}{4}}$

A negative power means take the reciprocal: $16^{-\frac{7}{4}} = (\frac{1}{16})^{\frac{7}{4}}$

The denominator 4 means the fourth root: the fourth root of $\frac{1}{16}$ is $\frac{1}{2}$

The numerator 7 means raise to the power 7: $(\frac{1}{2})^7 = \frac{1}{128}$

Answer: $\frac{1}{128}$

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6 Sample space and conditional probability

A fair 6-sided dice and a spinner numbered 1 to 5 are each thrown once. The first score and the second score are recorded.
(a) Work out the probability that the sum is greater than 5.
(b) Given that the first number is even, work out the probability that the sum is greater than 5.

(a) There are $6 \times 5 = 30$ equally likely outcomes, and 20 of them satisfy: the sum is greater than 5

Probability = $\frac{2}{3}$

(b) 15 outcomes satisfy: the first number is even. Of these, 11 also satisfy: the sum is greater than 5

Probability = $\frac{11}{15}$

Answer: (a) $\frac{2}{3}$ (b) $\frac{11}{15}$

7 Terms of a quadratic sequence

The nth term of a sequence is $n^2 + 2n + 7$.
(a) Work out the 10th term.
(b) Is 293 a term of the sequence? You must show your working.

(a) $10^2 + 2 \times 10 + 7 = 127$
(b) Solve $n^2 + 2n + 7 = 293$, so $n^2 + 2n - 286 = 0$
$b^2 - 4ac = 2^2 - 4 \times 1 \times (-286) = 1148$, which is not a square number
So n is not a whole number, and 293 is not a term
Answer: (a) 127 (b) No

8 Fractional indices

Given that $x > 0$, solve $x^{-\frac{3}{2}} = \frac{1}{64}$

Raise both sides to the power $-\frac{2}{3}$

$x = (\frac{1}{64})^{-\frac{2}{3}}$

$x = 16$

Answer: x = 16

9 Perpendicular bisector

A is the point $(-3, 0)$ and B is the point $(7, 8)$.
Find an equation of the perpendicular bisector of AB.
Give your answer in the form $ax + by = c$, where a, b and c are integers.

Midpoint of AB = $(\frac{-3+7}{2}, \frac{0+8}{2}) = (2, 4)$

Gradient of AB = $\frac{8-0}{7-(-3)} = \frac{4}{5}$

The perpendicular gradient is $-1 \div \frac{4}{5} = -\frac{5}{4}$

Using the midpoint: $y - 4 = -\frac{5}{4}(x - 2)$

Multiply by 4 and rearrange: $5x + 4y = 26$

Answer: 5x + 4y = 26

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10 Estimated mean with an unknown frequency

The table gives information about the masses of some eggs.
An estimate for the mean is 37.5.
Work out the value of x.

Mid-points: 15, 25, 35, 45, 55
Estimated mean = $\Sigma fm \div \Sigma f$, so $(1935 + 35x) \div (51 + x) = 37.5$
$1935 + 35x = 37.5(51 + x) = 1912.5 + 37.5x$
$(35 - 37.5)x = 1912.5 - 1935$
$-2.5x = -22.5$, so $x = 9$
Answer: x = 9