Day 242 **S Tier 10-a-day: Model Solutions**

1 Fractions

Work out $2\frac{1}{2} \div 1\frac{1}{2} + 2\frac{5}{8}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions

$\frac{5}{2} \div \frac{3}{2} = \frac{5}{2} \times \frac{2}{3} = \frac{5}{3}$

$\frac{5}{3} + \frac{21}{8} = \frac{103}{24}$

$= 4\frac{7}{24}$

Answer: $4\frac{7}{24}$

2 Cyclic quadrilaterals

A, B, C and D are points on a circle. Angle DAB = (3x + 30)°, angle BCD = (5x + 38)° and angle ABC = (2x + 30)°.
(a) Work out the value of x.
(b) Work out the size of angle ADC.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
(3x + 30) + (5x + 38) = 180
8x + 68 = 180, so 8x = 112
x = 14
(b) Angle ABC = 2 × 14 + 30 = 58°
Angle ADC = 180 − 58 = 122°
Answer: (a) x = 14 (b) 122°

3 Expanding brackets with surds

Expand and simplify $(2 + \sqrt{3})(6 - 3\sqrt{3})$

$(2 + \sqrt{3})(6 - 3\sqrt{3}) = 12 - 6\sqrt{3} + 6\sqrt{3} - 9$
$= 3$
Answer: 3

4 Arc length and sector area

A sector of a circle has angle 216° and area $\frac{363}{5}\pi$ cm^2.

(a) Work out the radius of the circle.
(b) Work out the length of the arc of the sector. Give your answer to (b) in terms of π.

(a) Area = $\frac{216}{360} \times \pi \times r^2$, so $\frac{216}{360} \times r^2 = \frac{363}{5}$

$r^2 = \frac{363}{5} \times \frac{360}{216} = 121$

r = 11 cm

(b) Arc length = $\frac{216}{360} \times 2 \times \pi \times 11 = \frac{66}{5}\pi$ cm

Answer: (a) 11 cm (b) $\frac{66}{5}\pi$ cm

5 Exchange rates

Carlos changes £1200 into euros at £1 = €1.15.
Carlos spends €520 and then changes the rest back into pounds at £1 = €1.25.
How many pounds does Carlos get back?

£1200 × 1.15 = €1380
1380 − 520 = €860
860 ÷ 1.25 = £688
Answer: £688

Day 242 **S Tier 10-a-day: Model Solutions**

6 Terms of a quadratic sequence

The nth term of a sequence is $n^2 + 7n - 6$.
(a) Work out the 12th term.
(b) Is 362 a term of the sequence? You must show your working.

(a) $12^2 + 7 \times 12 - 6 = 222$
(b) Solve $n^2 + 7n - 6 = 362$, so $n^2 + 7n - 368 = 0$
Factorise or use the formula: $n = 16$ or $n = -23$
$n = 16$ is a positive whole number, so 362 is term number 16
Answer: (a) 222 (b) Yes, it is the 16th term

7 Speed and a moving train

A train travels at a constant speed of 54 km/h.
It takes 60 seconds to cross completely over a bridge which is 700 m long.
Work out the length of the train, in metres.

Speed = 54 km/h = 54,000 ÷ 3600 = 15 m/s
Distance travelled in 60 s = 15 × 60 = 900 m
This is the length of the bridge plus the length of the train
Train = 900 − 700 = 200 m
Answer: 200 m

8 Comparing box plots

The box plots show the test marks of the students in two classes, A and B.
(a) Work out the interquartile range for each.
(b) Compare the distributions of A and B.

(a) IQR of A = 55 − 40 = 15
IQR of B = 85 − 35 = 50
(b) Class B has the higher median (60 compared with 45), so on average Class B scored higher
Class A has the smaller interquartile range (15 compared with 50), so its marks are more consistent
Answer: (a) A: 15, B: 50 (b) B has the higher median, A has the smaller IQR

9 Nested surds

Show that $\sqrt{28 + 10\sqrt{3}} = 5 + \sqrt{3}$

Square the right-hand side: $(5 + \sqrt{3})^2 = 25 + 10\sqrt{3} + 3$
$= 28 + 10\sqrt{3}$
So $\sqrt{28 + 10\sqrt{3}} = 5 + \sqrt{3}$
Answer: $\sqrt{28 + 10\sqrt{3}} = 5 + \sqrt{3}$

10 Tangency conditions

The line $y = kx - 3$ is a tangent to the curve $y = x^2 + 2$.
Find the possible values of k.
Give any surds in their simplest form.

Equate: $x^2 + 2 = kx - 3$
Rearrange: $x^2 - kx + 5 = 0$
One point of contact, so $b^2 - 4ac = 0$: $k^2 - 4 \times 5 = 0$
$k^2 = 20$
$k = \pm\sqrt{20} = \pm 2\sqrt{5}$
Answer: $k = \pm 2\sqrt{5}$