Day 241 **S Tier 10-a-day: Model Solutions**

1 Percentage change

A price is increased by 100%.
By what percentage must the new value be decreased to get back to the original value?

Let the original be 100. After a 100% increase it is 200.
To return to 100, decrease by 100

Percentage decrease = $\frac{100}{200} \times 100 = 50\%$

Answer: 50%

2 Capture and recapture

30 beetles in a field are caught, marked and put back.
Later, a second sample of 50 beetles is caught. 10 of these are marked.
(a) Work out an estimate for the number of beetles in a field.
(b) State one assumption you have made.

(a) The proportion marked in the second sample is 10 out of 50

This should match the proportion marked in the whole population: $\frac{10}{50} = \frac{30}{N}$

N = 30 × 50 ÷ 10 = 150
(b) No beetles are born, die or leave between the samples, and the beetles mix evenly
Answer: (a) 150 (b) The population stays the same between samples

3 Quadratic inequalities

Solve $2x^2 \leq 8$

Rearrange: $2x^2 - 8 \leq 0$
Divide by 2: $x^2 - 4 \leq 0$
Solve $x^2 - 4 = 0$: $(x + 2)(x - 2) = 0$, so $x = -2$ or $x = 2$
The graph is a U shape crossing the x-axis at −2 and 2; it is below the axis between the roots
$-2 \leq x \leq 2$
Answer: $-2 \leq x \leq 2$

4 Perpendicular bisector

A is the point (0, 5) and B is the point (−8, 1).
Find an equation of the perpendicular bisector of AB. Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = (−4, 3)

Gradient of AB = $\frac{1}{2}$

Perpendicular gradient = −2
y − 3 = (−2)(x − (−4))
Rearrange to integer coefficients: 2x + y = −5
Answer: 2x + y = −5

5 Exchange rates

Zane changes £1100 into euros at £1 = €1.2.
Zane spends €880 and then changes the rest back into pounds at £1 = €1.6.
How many pounds does Zane get back?

£1100 × 1.2 = €1320
1320 − 880 = €440
440 ÷ 1.6 = £275
Answer: £275

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6 Where a line meets the axes

The line L has equation $8x + 15y = -120$.
L crosses the x-axis at the point A and the y-axis at the point B.
(a) Find the coordinates of A and B.
(b) Work out the area of triangle OAB, where O is the origin.

(a) At A, $y = 0$: $8x = -120$, so $x = -15$. A is $(-15, 0)$
At B, $x = 0$: $15y = -120$, so $y = -8$. B is $(0, -8)$
(b) OA = 15 and OB = 8, and angle AOB = 90°

Area $= \frac{1}{2} \times 15 \times 8 = 60$

Answer: (a) A$(-15, 0)$, B$(0, -8)$ (b) 60

7 Comparing box plots

The box plots show the times taken by two groups, A and B.
(a) Work out the interquartile range for each.
(b) Compare the distributions of A and B.

(a) IQR of A $= 75 - 40 = 35$
IQR of B $= 90 - 30 = 60$
(b) Group B has the lower median (40 compared with 60), so on average Group B was faster
Group A has the smaller interquartile range (35 compared with 60), so its times are more consistent
Answer: (a) A: 35, B: 60 (b) B has the lower median, A has the smaller IQR

8 Tangent to a circle

A circle has equation $x^2 + y^2 = 625$.
The point P$(24, -7)$ lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form $ax + by = c$, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $-\frac{7}{24}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div (-\frac{7}{24}) = \frac{24}{7}$

$y - (-7) = \frac{24}{7}(x - 24)$

Multiply out and rearrange: $24x - 7y = 625$
(b) On the x-axis $y = 0$, so $24x = 625$
$x = \frac{625}{24}$, so T is $(\frac{625}{24}, 0)$

Answer: (a) $24x - 7y = 625$ (b) T $= (\frac{625}{24}, 0)$

9 Exact lengths and areas

A right-angled triangle has base $(7 + \sqrt{6})$ cm and height $(7 - \sqrt{6})$ cm.
Show that the area of the triangle is a rational number.
Work out the area.

Area $= \frac{1}{2} \times (7 + \sqrt{6})(7 - \sqrt{6})$

$(7 + \sqrt{6})(7 - \sqrt{6}) = 49 - 6 = 43$
Area $= \frac{1}{2} \times 43 = \frac{43}{2}$

Answer: $\frac{43}{2}$ cm^2

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10 Solving trigonometric equations

Solve $5 \sin x - 2 = 0$ for $0° \leq x \leq 360°$. (Calculator)
Give your answers correct to 1 decimal place.

$\sin x = \frac{2}{5}$

Related acute angle: $\sin^{-1}(\frac{2}{5}) = 23.6°$

sine is positive in the 1st and 2nd quadrants, so $x = 23.6°$ and $x = 180° - 23.6°$
$x = 23.6°$ and $x = 156.4°$
Answer: x = 23.6° or x = 156.4°