Day 240 **S Tier 10-a-day: Model Solutions**

1 Mean of a changing set

The mean of 11 numbers is 100.
A new number is added and the mean of all 12 numbers is 93.
Work out the new number.

Total of the 11 numbers = 11 × 100 = 1100
New total = 12 × 93 = 1116
New number = 1116 − 1100 = 16
Answer: 16

2 Fractions

Work out $2\frac{1}{3} \times 2\frac{5}{8} - 4\frac{1}{2}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions

$\frac{7}{3} \times \frac{21}{8} = \frac{49}{8}$

$\frac{49}{8} - \frac{9}{2} = \frac{13}{8}$

$= 1\frac{5}{8}$

Answer: $1\frac{5}{8}$

3 Quadratic formula and surds

Solve $4x^2 - 2x - 3 = 0$ (Calculator)
Give your answers correct to 2 decimal places.

Use the quadratic formula with a = 4, b = −2, c = −3

$x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 4 \times (-3)}}{2 \times 4} = \frac{2 \pm \sqrt{52}}{8}$

x = −0.6514... or x = 1.1514...
x = −0.65 or x = 1.15 (2 d.p.)
Answer: x = −0.65 or x = 1.15

4 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 12 cm, DB = 8 cm and the area of triangle ADE is 54 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 12 + 8 = 20 cm

Length scale factor = 20 ÷ 12 = $\frac{5}{3}$

Area scale factor = $(\frac{5}{3})^2 = \frac{25}{9}$

Area of triangle ABC = $54 \times \frac{25}{9} = 150$ cm^2

Trapezium = 150 − 54 = 96 cm^2

Answer: 96 cm^2

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5 Direct proportion formulae

E is directly proportional to v.
When v = 1, E = 12.
(a) Find a formula for E in terms of v.
(b) Work out the value of E when v = 4.

(a) $E = kv$
$12 = k \times 1$, so $k = 12 \div 1 = 12$
$E = 12v$
(b) $E = 12 \times 4 = 48$
Answer: (a) E = 12v (b) 48

6 Pythagoras with algebra

ABC is a right-angled triangle with sides (x + 3) cm, (x − 4) cm and (2x − 5) cm. The longest side is (2x − 5) cm.
(a) Find the value of x.
(b) Work out the area of the triangle.
The diagram is not drawn accurately.

(a) Pythagoras: $(x + 3)^2 + (x - 4)^2 = (2x - 5)^2$
Expand and collect terms: $2x^2 - 18x = 0$
Factorise: $2x(x - 9) = 0$
x = 9 or x = 0, and x must be positive, so x = 9
(b) The two shorter sides are 12 cm and 5 cm
Area = $\frac{1}{2} \times 12 \times 5 = 30$ cm^2
Answer: (a) x = 9 (b) 30 cm^2

7 Fractional and negative indices

Work out $(\frac{125}{27})^{-\frac{7}{3}}$

A negative power means take the reciprocal: $(\frac{125}{27})^{-\frac{7}{3}} = (\frac{27}{125})^{\frac{7}{3}}$
The denominator 3 means the cube root: the cube root of $\frac{27}{125}$ is $\frac{3}{5}$
The numerator 7 means raise to the power 7: $(\frac{3}{5})^7 = \frac{2187}{78125}$
Answer: $\frac{2187}{78125}$

8 Index equations

Solve $2^{4x+4} = 4^{x+4}$

Write both sides as powers of 2: $4 = 2^2$
Equate the powers: $4x + 4 = 2x + 8$
$2x = 4$, so $x = 2$
Answer: x = 2

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9 Geometric sequences

The 2nd term of a geometric sequence is 24.
The 5th term of the sequence is 81.
(a) Work out the common ratio.
(b) Work out the first term.

(a) The common ratio r satisfies $r^3 = 81 \div 24 = \frac{27}{8}$

$r = \sqrt[3]{\frac{27}{8}} = \frac{3}{2}$

(b) The 2nd term is $a \times r^1$: $a \times \frac{3}{2} = 24$

$a = 24 \div \frac{3}{2} = 16$

Answer: (a) r = $\frac{3}{2}$ (b) 16

10 Exact trigonometric values

Work out the exact value of $(\tan 60°)^2 + \tan 45°$
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$(\tan 60°)^2 + \tan 45° = (\sqrt{3})^2 + 1$
$= 4$
Answer: 4