Day 24 **S Tier 10-a-day: Model Solutions**

1 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = (4x − 15)°
Angle PQR = (3x − 15)°
Angle PRS = (6x − 3)°
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
6x − 3 = 7x − 30
x = 27
(b) Angle PRS = 159° so angle PRQ = 180° − 159° = 21°
Answer: (a) x = 27 (b) 21°

2 Fractional and negative indices

Work out $27^{\frac{7}{3}}$

The denominator 3 means the cube root: the cube root of 27 is 3
The numerator 7 means raise to the power 7: $(3)^7 = 2187$
Answer: 2187

3 Inverse proportion in context

24 machines can make a batch of bolts in 20 hours.
After 2 hours, 8 of the machines leave. The rest carry on working at the same rate.
How many more hours does it take to finish?

Total work = 24 × 20 = 480 machine-hours
Work done = 24 × 2 = 48, so work left = 432
Workers left = 24 − 8 = 16
Time = 432 ÷ 16 = 27 hours
Answer: 27 hours

4 Solving trigonometric equations

Solve 5 cos x + 2 = 0 for 0° ≤ x ≤ 360°. (Calculator)
Give your answers correct to 1 decimal place.

$\cos x = -\frac{2}{5}$

Related acute angle: $\cos^{-1}(\frac{2}{5}) = 66.4°$

cosine is negative in the 2nd and 3rd quadrants, so x = 180° − 66.4° and x = 180° + 66.4°
x = 113.6° and x = 246.4°
Answer: x = 113.6° or x = 246.4°

5 Substitution

Work out the value of $3a - b^3$
when a = −2, b = −1 and c = 1.

Substitute the values: $3 \times (-2) - (-1)^3$
= −6 − (−1)
= −5
Answer: −5

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6 At least one

The probability that a light bulb in a hallway fails in any month is $\frac{3}{10}$.
Events on different occasions are independent.
Work out the probability that this happens at least once in 6 months.
Give your answer correct to 3 significant figures. (Calculator)

P(never) = $(1 - \frac{3}{10})^6 = \frac{7}{10}^6 = 0.117649$
P(at least once) = 1 − P(never) = 1 − 0.117649
= 0.882
Answer: 0.882

7 Trigonometric graphs

The diagram shows the graph of $y = a \sin x° + c$ for $0 \leq x \leq 360$.
Some points on the graph are labelled.
Find the value of a and the value of c.

At x = 90, sin x° = 1, so a + c = −3
At x = 270, sin x° = −1, so −a + c = −5
Add: 2c = −8, so c = −4
Subtract: 2a = 2, so a = 1
Answer: a = 1, c = −4

8 Reverse percentage growth and decay

A laptop loses 15% of its value each year.
After 3 years it is worth £17,195.50.
Work out its value when it was new. (Calculator)

Yearly multiplier = 0.85
Original × 0.85^3 = £17,195.50
Original = £17,195.50 ÷ 0.85^3 = £28,000
Answer: £28,000

9 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 3 cm, top radius 2 cm and base radius 4 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+3} = \frac{2}{4}$
4 h = 2(h + 3), so 2 h = 6 and h = 3
Large cone: $\frac{1}{3} \times \pi \times 4^2 \times 6 = 32\pi$
Small cone: $\frac{1}{3} \times \pi \times 2^2 \times 3 = 4\pi$
Frustum = 32π − 4π = 28π cm^3
Answer: 28π cm^3

10 Counting factors

$N = 2^2 \times 5^3$
How many factors does N have?
Include 1 and N.

Add 1 to each power and multiply
(2 + 1) × (3 + 1) = 3 × 4 = 12
Answer: 12