Day 239 **S Tier 10-a-day: Model Solutions**

1 Factorising quadratics

Factorise $6x^2 - 7x - 20$

$6 \times (-20) = -120$. Two numbers that multiply to -120 and add to -7 are -15 and 8

Split the middle term: $6x^2 - 15x + 8x - 20$

Factorise in pairs: $3x(2x - 5) + 4(2x - 5)$

$= (3x + 4)(2x - 5)$

Answer: $(3x + 4)(2x - 5)$

2 Hollow cylinder and density

A metal pipe is a hollow cylinder of length 50 cm. (Calculator)

The outer radius is 4 cm and the inner radius is 2.5 cm.

The density of the metal is 7.8 g/cm^3.

Work out the mass of the pipe. Give your answer in kilograms correct to 3 significant figures.

Volume of metal $= \pi \times 4^2 \times 50 - \pi \times 2.5^2 \times 50 = \pi(16 - 6.25) \times 50$

$= 1531.5$ cm^3

Mass = density × volume $= 7.8 \times 1531.5 = 11946$ g

$= 11.9$ kg (3 s.f.)

Answer: 11.9 kg

3 Percentage change

The population of a village increased by 60% in 2023.

In 2024 the population increased by a further 30%.

Work out the overall percentage increase over the two years.

Multipliers: 1.6 and 1.3

Overall multiplier $= 1.6 \times 1.3 = 2.08$

$2.08 - 1 = 1.08$, so an overall increase of 108%

Answer: 108% increase

4 Quadratic formula and surds

Solve $4x^2 - 7x - 4 = 0$

Give your answers in surd form.

Use the quadratic formula with $a = 4$, $b = -7$, $c = -4$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4 \times 4 \times (-4)}}{2 \times 4}$$

$$x = \frac{7 \pm \sqrt{113}}{8}$$

Answer: $x = \frac{7 \pm \sqrt{113}}{8}$

5 Area of a triangle using sine

Triangle ABC has AB = 16 cm and BC = 6 cm.

The area of triangle ABC is $24\sqrt{2}$ cm^2.

Angle ABC is obtuse.

Work out the size of angle ABC.

Do not use a calculator.

$$\text{Area} = \frac{1}{2} \times AB \times BC \times \sin B$$

$$24\sqrt{2} = \frac{1}{2} \times 16 \times 6 \times \sin B$$

$$\sin B = 24\sqrt{2} \div 48 = \frac{\sqrt{2}}{2}$$

$\sin 45° = \frac{\sqrt{2}}{2}$, and B is obtuse, so $B = 180° - 45° = 135°$

Answer: 135°

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6 Rationalising the denominator

Write $\frac{15}{2+\sqrt{2}}$ in the form $\frac{p+q\sqrt{2}}{r}$
where p, q and r are integers and r > 1,
and the fraction is in its simplest form.

Multiply top and bottom by $(2-\sqrt{2})$:

$$\frac{15}{2+\sqrt{2}} = \frac{15(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})}$$

Denominator: $4-2=2$

$$= \frac{30-15\sqrt{2}}{2}$$

Answer: $\mathbf{\frac{30-15\sqrt{2}}{2}}$

7 At least one

The probability that the 8:15 train is late on any day is $\frac{1}{10}$.

Events on different occasions are independent.
Work out the probability that this happens at least once in 4 days.
Give your answer correct to 3 significant figures. (Calculator)

$P(\text{never}) = (1-\frac{1}{10})^4 = \frac{9}{10}^4 = 0.6561$

$P(\text{at least once}) = 1 - P(\text{never}) = 1 - 0.6561$
$= 0.344$

Answer: 0.344

8 Inverse functions

$f(x) = \frac{2}{x+4} - 1$

(a) Find $f^{-1}(x)$.
(b) Find the value of $f^{-1}(0)$.

(a) Let $y = \frac{2}{x+4} - 1$, so $y+1 = \frac{2}{x+4}$

Make x the subject: $x+4 = \frac{2}{y+1}$

$x = \frac{2}{y+1} - 4$

$f^{-1}(x) = \frac{2}{x+1} - 4$

(b) $f^{-1}(0) = \frac{2}{0-(-1)} - 4 = -2$

Answer: (a) $\mathbf{f^{-1}(x) = \frac{2}{x+1} - 4}$ **(b)** $\mathbf{-2}$

9 Exchange rates

Rhys changes £300 into euros at £1 = €1.18.
Rhys spends €220 and then changes the rest back into pounds at £1 = €1.6.
How many pounds does Rhys get back?

£300 × 1.18 = €354
354 − 220 = €134
134 ÷ 1.6 = £83.75

Answer: £83.75

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10 Loci and regions

ABCD is a rectangle with AB = 10 cm and AD = 11 cm.
A point P inside the rectangle is such that AP ≤ 10 cm and P is nearer to A than to B.
Work out the area of the region in which P can lie. Give your answer in terms of π and surds.

Nearer to A than to B: P is on the A side of the perpendicular bisector of AB, a distance 5 cm from A
AP ≤ 10 cm: P is inside the quarter circle of radius 10 cm centred at A

The bisector meets the arc where $\cos\theta = \frac{5}{10}$, so $\theta = 60°$ at A. The region is a right-angled triangle plus a sector of angle 30°

Triangle: base 5 cm, height $10\sin 60° = 5\sqrt{3}$ cm, area $= \frac{1}{2} \times 5 \times 5\sqrt{3} = \frac{25}{2}\sqrt{3}$

Sector of angle 30° : $\frac{30}{360} \times \pi \times 10^2 = \frac{25}{3}\pi$

Area $= \frac{25}{2}\sqrt{3} + \frac{25}{3}\pi$ cm^2

Answer: $\left(\frac{25}{2}\sqrt{3} + \frac{25}{3}\pi\right)$ cm^2