Day 238 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

The interior angles of a pentagon are
$x°$, $(2x - 30)°$, $(x + 15)°$, $(x - 5)°$, $(x + 20)°$
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = $(5 - 2) \times 180 = 540°$
$6x = 540$, so $6x = 540$
$x = 90$
(b) The angles are 90°, 150°, 105°, 85°, 110°
The largest is 150°
Answer: (a) x = 90 (b) 150°

2 Solving quadratics by factorising

Solve $x^2 + 3x - 18 = 0$

Factorise: $(x + 6)(x - 3) = 0$
$x + 6 = 0$ or $x - 3 = 0$
$x = -6$ or $x = 3$
Answer: x = −6 or x = 3

3 Tests of divisibility

The 5-digit number 20k20 is a multiple of 9.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 9 if its digit sum is a multiple of 9
Digit sum = 2 + 0 + 2 + 0 + k = 4 + k
This must be a multiple of 9, so k = 5
Answer: k = 5

4 Terms of a quadratic sequence

The nth term of a sequence is $n^2 + 7n - 6$.
(a) Work out the 8th term.
(b) Is 54 a term of the sequence? You must show your working.

(a) $8^2 + 7 \times 8 - 6 = 114$
(b) Solve $n^2 + 7n - 6 = 54$, so $n^2 + 7n - 60 = 0$
Factorise or use the formula: $n = 5$ or $n = -12$
n = 5 is a positive whole number, so 54 is term number 5
Answer: (a) 114 (b) Yes, it is the 5th term

5 Compound interest

Evan invests £3,000.
The account pays compound interest of 4% a year for the first 2 years,
then 1% a year for the next 2 years.
Work out the total amount after 4 years, to the nearest penny. (Calculator)

Multipliers: 1.04 for 2 years, then 1.01 for 2 years
$£3{,}000 \times 1.04^2 \times 1.01^2$
= £3,310.02
Answer: £3,310.02

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6 Exponential growth and decay

A population of fish starts at 1,000 and grows by 20% each year.
After how many whole years will the population first be at least double its starting value? (Calculator)

Multiplier = 1.2, double is 2,000

After 3 years: $1000 \times 1.2^3 = 1728.0$

After 4 years: $1000 \times 1.2^4 = 2073.6$

Answer: 4 years

7 Equations with indices

Solve $x^{\frac{3}{2}} = 125$

Write 125 as a power of 5: $125 = 5^3$

$x^{\frac{3}{2}} = 5^3$, so $x = (5^3)^{\frac{2}{3}}$

$x = 5^2 = 25$

Answer: x = 25

8 Speed and a moving train

A train travels at a constant speed of 90 km/h.
It takes 30 seconds to cross completely over a bridge which is 430 m long.
Work out the length of the train, in metres.

Speed = 90 km/h = 90,000 ÷ 3600 = 25 m/s
Distance travelled in 30 s = 25 × 30 = 750 m
This is the length of the bridge plus the length of the train
Train = 750 − 430 = 320 m

Answer: 320 m

9 Tangent to a circle

A circle has equation $x^2 + y^2 = 841$.
The point P(20, −21) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $-\frac{21}{20}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div (-\frac{21}{20}) = \frac{20}{21}$

$y - (-21) = \frac{20}{21}(x - 20)$

Multiply out and rearrange: $20x - 21y = 841$

(b) On the x-axis y = 0, so 20x = 841

$x = \frac{841}{20}$, so T is $(\frac{841}{20}, 0)$

Answer: (a) 20x − 21y = 841 (b) T = $(\frac{841}{20}, 0)$

10 Rationalising the denominator

Write $\frac{5 + \sqrt{10}}{5 - \sqrt{10}}$ in the form $p + q\sqrt{10}$
where p and q are rational numbers.

Multiply top and bottom by $(5 + \sqrt{10})$:

Numerator: $(5 + \sqrt{10})(5 + \sqrt{10}) = 35 + 10\sqrt{10}$

Denominator: 25 − 10 = 15

$= \frac{7 + 2\sqrt{10}}{3}$

Answer: $\frac{7 + 2\sqrt{10}}{3}$