Day 237 **S Tier 10-a-day: Model Solutions**

1 Three-part ratios

Amir, Fatima and Tariq share some sweets.
Amir : Fatima = 4 : 9
Fatima : Tariq = 6 : 2
(a) Work out the ratio Amir : Fatima : Tariq.
(b) Fatima gets 42 sweets more than Tariq. How many sweets does Tariq get?

(a) Make the Fatima part the same: 4 : 9 = 8 : 18 and 6 : 2 = 18 : 6
Amir : Fatima : Tariq = 8 : 18 : 6 = 4 : 9 : 3
(b) Difference in parts = 6, so 1 part = 42 ÷ 6 = 7
Tariq gets 3 × 7 = 21 sweets
Answer: (a) 4 : 9 : 3 (b) 21 sweets

2 Tangent to a circle

A circle has equation $(x - 4)^2 + (y - 2)^2 = 65$.
The point P(0, 9) lies on the circle.
Find an equation of the tangent to the circle at P.
Give your answer in the form ax + by = c, where a, b and c are integers.

The centre is (4, 2). The gradient of the radius to P is $\frac{9-2}{0-4} = -\frac{7}{4}$

The tangent is perpendicular to the radius, so its gradient is $\frac{4}{7}$. Using P: $y - 9 = \frac{4}{7}(x)$

Multiply by 7: $7(y - 9) = 4(x)$
Expand and rearrange: $-4x + 7y = 63$
Multiply by -1: $4x - 7y = -63$
Answer: $4x - 7y = -63$

3 Weighted percentages

The ratio of juniors to seniors in a school is 3 : 5.
75% of the juniors and 15% of the seniors play a sport.
What percentage of all of these students play a sport?

Imagine 160 students in total: 60 juniors and 100 seniors
75% of 60 = 45 and 15% of 100 = 15
Together: 60 out of 160
60 ÷ 160 × 100 = 37.5%
Answer: 37.5%

4 Rationalising the denominator

Rationalise the denominator of $\frac{13}{3\sqrt{3}}$
Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{3}$:
$\frac{13}{3\sqrt{3}} = \frac{13\sqrt{3}}{9}$

Answer: $\frac{13\sqrt{3}}{9}$

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5 Direct and inverse proportion

T is directly proportional to the square root of l.
When l = 36, T = 12.
(a) Find a formula for T in terms of l.
(b) Find the value of T when l = 4.

(a) $T = k\sqrt{l}$
Substitute l = 36 and T = 12: $12 = k \times \sqrt{36}$
k = 2
$T = 2\sqrt{l}$
(b) $T = 2\sqrt{4} = 4$
Answer: (a) $T = 2\sqrt{l}$ (b) 4

6 Transformations of graphs

The graph of y = f(x) has a minimum point at (1, −4).
Write down the coordinates of the turning point of the graph of
(a) y = f(x) + 3
(b) y = f(x + 3) − 5

(a) Start with the turning point (1, −4)
Apply the transformation to the point: (1, −1)
(b) f(x + 3) changes the x-coordinate to −2, and f(x) − 5 changes the y-coordinate to −9
Turning point: (−2, −9)
Answer: (a) (1, −1) (b) (−2, −9)

7 Income tax

In one country, income tax is worked out like this:
The first £15,000 of income is not taxed.
The next £30,000 is taxed at 20%.
All income above that is taxed at 45%.
Sofia pays £18,375 in income tax in a year.
Work out Sofia's income for the year.

Tax on the 20% band = 20% of £30,000 = £6,000
Tax paid at 45% = £18,375 − £6,000 = £12,375
Income taxed at 45% = £12,375 ÷ 0.45 = £27,500
Income = £15,000 + £30,000 + £27,500 = £72,500
Answer: £72,500

8 Reverse percentages

The price of a games console is increased by 15%.
Later the new price is reduced by 15% in a sale.
The sale price is £156.40.
Work out the price before the increase. (Calculator)

Multipliers: 1.15 and 0.85
Overall multiplier = 1.15 × 0.85 = 0.9775
Original × 0.9775 = £156.40
Original = £156.40 ÷ 0.9775 = £160
Answer: £160

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9 Exact lengths and areas

A rectangle has area $(7 + 4\sqrt{2})$ cm^2 and length $(3 + 2\sqrt{2})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{2}$

Width = area ÷ length = $\dfrac{7 + 4\sqrt{2}}{3 + 2\sqrt{2}}$

Multiply top and bottom by $(3 - 2\sqrt{2})$:
Numerator: $(7 + 4\sqrt{2})(3 - 2\sqrt{2}) = 5 - 2\sqrt{2}$
Denominator: $9 - 8 = 1$
Width = $5 - 2\sqrt{2}$
Answer: $5 - 2\sqrt{2}$ cm

10 Hollow cylinder and density

A metal pipe is a hollow cylinder with outer radius 8 cm and inner radius 7.5 cm. (Calculator)
The density of the metal is 8.9 g/cm^3.
The mass of the pipe is 16.3 kg.
Work out the length of the pipe. Give your answer in centimetres correct to 3 significant figures.

Mass = 16300 g, so volume = mass ÷ density = 16300 ÷ 8.9 = 1831.5 cm^3
Cross-sectional area = $\pi \times 8^2 - \pi \times 7.5^2 = 24.347$ cm^2
Length = volume ÷ area = 1831.5 ÷ 24.347 = 75.2 cm (3 s.f.)
Answer: 75.2 cm