Day 236 **S Tier 10-a-day: Model Solutions**

1 Factorising quadratics

Factorise $12x^2 + 29x + 14$

$12 \times 14 = 168$. Two numbers that multiply to 168 and add to 29 are 21 and 8

Split the middle term: $12x^2 + 21x + 8x + 14$

Factorise in pairs: $3x(4x + 7) + 2(4x + 7)$

$= (3x + 2)(4x + 7)$

Answer: (3x + 2)(4x + 7)

2 Inverse proportion in context

12 workers can build a wall in 15 days.
After 3 days, 4 of the workers leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work = $12 \times 15 = 180$ worker-days

Work done = $12 \times 3 = 36$, so work left = 144

Workers left = $12 - 4 = 8$

Time = $144 \div 8 = 18$ days

Answer: 18 days

3 Transformations of graphs

The point P(−4, −6) lies on the graph of y = f(x).
Write down the coordinates of the image of P on the graph of
(a) y = f(x + 5) (b) y = f(−x)

(a) y = f(x + 5) is a translation of 5 to the left: (−9, −6)

(b) y = f(−x) is a reflection in the y-axis, so the x coordinate changes sign: (4, −6)

Answer: (a) (−9, −6) (b) (4, −6)

4 Sine rule

The diagram shows triangle ABC. (Calculator)
Angle BAC = 34° and angle ABC is acute.
Work out the size of angle ABC, marked θ. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{\sin\theta}{AC} = \frac{\sin A}{BC}$

$\frac{\sin\theta}{7} = \frac{\sin 34°}{10}$

$\sin\theta = \frac{7 \times \sin 34°}{10} = 0.39144$

$\theta = \sin^{-1}(0.39144) = 23.0°$ (1 d.p.)

Answer: θ = 23.0°

5 Rates of work

Tap A can fill an empty tank in 20 minutes.
Tap B can fill an empty tank in 30 minutes.
Both are turned on at the same time.
How long do they take to fill an empty tank?

In 1 minute: Tap A does $\frac{1}{20}$ and Tap B does $\frac{1}{30}$

Together: $\frac{1}{20} + \frac{1}{30} = \frac{50}{600}$

Time = $\frac{600}{50}$ = 12 minutes

Answer: 12 minutes

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6 Equations with indices

Solve $x^{-\frac{3}{2}} = \frac{1}{125}$

Write $\frac{1}{125}$ as a power of 5: $\frac{1}{125} = 5^{-3}$

$x^{-\frac{3}{2}} = 5^{-3}$, so $x = (5^{-3})^{-\frac{2}{3}}$

$x = 5^2 = 25$

Answer: x = 25

7 Percentages of percentages

In a group of employees, 35% are men and the rest are women.
45% of the men and 30% of the women work from home.
What percentage of all the employees work from home?

Take 100 employees: 35 are men and 65 are women
45% of 35 = 15.75 and 30% of 65 = 19.5
15.75 + 19.5 = 35.25 out of 100
Answer: 35.25%

8 Tests of divisibility

The 4-digit number 107k is a multiple of 4.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 4 if its last two digits form a multiple of 4
Last two digits: 7k
Multiples of 4 of this form: 72, 76
Answer: k = 2 or 6

9 Combined transformations

The point P(−3, −1) is reflected in the line y = x.
The image is then reflected in the x-axis.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (−1, −3)
After the second transformation: P'' = (−1, 3)
(b) Follow a general point (x, y) through both transformations: (x, y) → (y, x) → (y, −x)
The single mapping is (x, y) → (y, −x)
This is a rotation 90° clockwise about the origin
Answer: (a) (−1, 3) (b) Rotation 90° clockwise about the origin

10 Tree diagrams and reverse probability

The probability that it rains on a day is 0.4.
If it rains, the probability that Ali is late for school is 0.7.
If it does not rain, the probability that Ali is late is 0.4.
(a) Work out the probability that Ali is late.
(b) Ali is late. Work out the probability that it rained.

(a) P(late) = 0.4 × 0.7 + 0.6 × 0.4 = 0.28 + 0.24 = 0.52
(b) P(rain and late) = 0.28

P(rain | late) = 0.28 ÷ 0.52 = $\frac{7}{13}$

Answer: (a) 0.52 (b) $\frac{7}{13}$