Day 235 **S Tier 10-a-day: Model Solutions**

1 Factorising quadratics

Factorise $3x^2 - 19x + 6$

$3 \times 6 = 18$. Two numbers that multiply to 18 and add to -19 are -18 and -1

Split the middle term: $3x^2 - 18x - x + 6$

Factorise in pairs: $3x(x - 6) - (x - 6)$

$= (3x - 1)(x - 6)$

Answer: $(3x - 1)(x - 6)$

2 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle OAB = 25°.

(a) Work out the size of angle AOB.

(b) Work out the size of angle ACB.

The diagram is not drawn accurately.

(a) OA = OB (radii), so triangle OAB is isosceles and angle OBA = 25°

Angle AOB = $180 - 2 \times 25 = 130°$

(b) The angle at the circumference is half the angle at the centre: $130 \div 2 = 65°$

Answer: (a) 130° (b) 65°

3 Fraction problems

A bag contains balloons.

$\frac{2}{5}$ of the balloons are pink and $\frac{3}{8}$ of the balloons are silver.

The rest of the balloons, 9 of them, are gold.

How many pink balloons are there?

Fraction that are gold = $1 - \frac{2}{5} - \frac{3}{8} = \frac{9}{40}$

$\frac{9}{40}$ of the total is 9

Total = $9 \div \frac{9}{40} = 40$

Pink balloons = $\frac{2}{5}$ of 40 = 16

Answer: 16

4 Quadratic graphs

The curve $y = x^2 - 8x - 9$ crosses the x-axis at the points A and B.

(a) Find the coordinates of A and B.

(b) Find the coordinates of the minimum point of the curve.

(a) Put $y = 0$: $x^2 - 8x - 9 = 0$, so $(x + 1)(x - 9) = 0$

$x + 1 = 0$ or $x - 9 = 0$, so $x = -1$ or $x = 9$

(b) The turning point is halfway between A and B: $x = (-1 + 9) \div 2 = 4$

When $x = 4$: $y = 4^2 - 8 \times 4 - 9 = -25$

Answer: (a) A(−1, 0), B(9, 0) (b) (4, −25)

5 Using a line of best fit

The scatter graph shows age of car (x) and value (y).

The equation of the line of best fit is $y = -0.5x + 24$.

(a) Estimate y when x = 3.

(b) Explain why an estimate of y when x = 25 may be unreliable.

(a) $y = -0.5 \times 3 + 24 = 22.5$

(b) x = 25 is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)

Answer: (a) 22.5 (b) x = 25 is outside the range of the data

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6 Vectors in a triangle

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 2 : 1. N is on AB with AN : NB = 1 : 2.
(a) Find ON in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find MN in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{1}{3}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a}) = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$

(b) OM = $\frac{2}{3}\mathbf{a}$

MN = ON − OM = $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} - \frac{2}{3}\mathbf{a} = \frac{1}{3}\mathbf{b}$

Answer: (a) $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ (b) $\frac{1}{3}\mathbf{b}$

7 Density of an alloy

An alloy contains 113 g of lead and 284 g of zinc.
The density of lead is 11.3 g/cm^3. The density of zinc is 7.1 g/cm^3.
Work out the density of the alloy.

Volume of lead = 113 ÷ 11.3 = 10 cm^3
Volume of zinc = 284 ÷ 7.1 = 40 cm^3
Total mass = 397 g, total volume = 50 cm^3
Density = 397 ÷ 50 = 7.94 g/cm^3
Answer: 7.94 g/cm^3

8 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $-3\mathbf{a} - 2\mathbf{b}$, OQ = $-5\mathbf{a} + 8\mathbf{b}$ and OR = $-8\mathbf{a} + 23\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $-2\mathbf{a} + 10\mathbf{b}$
QR = OR − OQ = $-3\mathbf{a} + 15\mathbf{b}$

QR = $\frac{3}{2}$ PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line
(b) PQ : QR = 2 : 3
Answer: (b) 2 : 3

9 Equal roots and real roots

Show that the equation $x^2 + 2kx - 1 = 0$ has two distinct real solutions for all values of k.

$b^2 - 4ac = (2k)^2 - 4 \times 1 \times (-1)$
$= 4k^2 + 4$
$4k^2$ is never negative, so $4k^2 + 4$ is always at least 4, which is greater than 0
So $b^2 - 4ac > 0$ for all k, and there are always two distinct real solutions
Answer: Shown

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10 Surds with unknowns

Given that $(a + \sqrt{15})(b + \sqrt{15}) = 21 + 5\sqrt{15}$,
where a and b are positive integers with a < b, find the values of a and b.

$(a + \sqrt{15})(b + \sqrt{15}) = ab + (a + b)\sqrt{15} + 15$

Compare the $\sqrt{15}$ terms: $a + b = 5$

Compare the integer terms: $ab + 15 = 21$, so $ab = 6$

Two numbers with sum 5 and product 6 are 2 and 3

Answer: a = 2, b = 3