Day 234 **S Tier 10-a-day: Model Solutions**

1 Mean of a changing set

The mean of 11 numbers is 79.
One number is removed and the mean of the remaining 10 numbers is 86.3.
Work out the number that was removed.

Total of the 11 numbers = 11 × 79 = 869
Total of the 10 remaining = 10 × 86.3 = 863
Number removed = 869 − 863 = 6
Answer: 6

2 Surds

Given that $\sqrt{7} = \sqrt{n}$,
find the value of n.

$\sqrt{7} = \sqrt{1} \times \sqrt{7} = \sqrt{7}$
n = 7
Answer: n = 7

3 Simplifying algebraic fractions

Simplify fully $\frac{5x^2 - 125}{x^2 - 9x + 20}$

Factorise the top: $5(x - 5)(x + 5)$
Factorise the bottom: $(x - 5)(x - 4)$
Cancel the common factor $(x - 5)$: $\frac{5(x + 5)}{x - 4}$

Answer: $\mathbf{\frac{5(x + 5)}{x - 4}}$

4 LCM problems

A red light flashes every 7 minutes.
A blue light flashes every 9 minutes.
A green light flashes every 21 minutes.
They all flash together at 7:00 am.
How many times do they all flash together between 7:00 am and 3:00 pm, including both 7:00 am and 3:00 pm if they do?

Find the LCM of 7, 9 and 21
$7 = 7$, $9 = 3^2$, $21 = 3 \times 7$
LCM = $3^2 \times 7 = 63$
Total time from 7:00 am to 3:00 pm = 480 minutes
480 ÷ 63 = 7.62, so there are 7 more times after the start
Including the start: 7 + 1 = 8
Answer: 8

5 Cosine rule and area of a triangle

The diagram shows triangle ABC. (Calculator)
(a) Work out the length of AB, marked x, to 1 decimal place.
(b) Work out the area of triangle ABC to 3 significant figures.
The diagram is not drawn accurately.

(a) $x^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \times \cos 50°$
$x^2 = 38.292$
x = 6.2 cm (1 d.p.)

(b) Area = $\frac{1}{2} \times 6 \times 8 \times \sin 50° = 18.4$ cm^2 (3 s.f.)

Answer: (a) 6.2 cm (b) 18.4 cm^2

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6 Average speed

A train travels the first half of a journey at 50 km/h.
It travels the second half of the same journey at 75 km/h.
Work out the average speed for the whole journey.

Let each half of the journey be 150 km (a common multiple of 50 and 75)
Time for first half = 150 ÷ 50 = 3 hours
Time for second half = 150 ÷ 75 = 2 hours
Average speed = total distance ÷ total time = 300 ÷ 5 = 60 km/h
Answer: 60 km/h

7 Growth and decay

A rare car is worth £1,200.
Its value increases by 10% each year.
After how many whole years will it first be worth more than £2,100? (Calculator)

Multiplier each year = 1.1
After 5 years: £1,200 × 1.1^5 = £1,932.61
After 6 years: £1,200 × 1.1^6 = £2,125.87
So the first time it is more than £2,100 is after 6 years
Answer: 6 years

8 Combining means

The mean of the heights in cm for the boys is 45.
The mean of the heights in cm for the girls is 41.
The mean for all of the boys and girls together is 42.6.
Work out the ratio of the number of boys to the number of girls.

Let there be p boys and q girls
Total = 45p + 41q, and the overall mean gives 45p + 41q = 42.6(p + q)
2.4p = 1.6q
p : q = 1.6 : 2.4 = 2 : 3
Answer: 2 : 3

9 Loci and regions

ABCD is a rectangle with AB = 4 cm and AD = 9 cm.
A point P inside the rectangle is such that AP ≤ 4 cm and P is nearer to A than to B.
Work out the area of the region in which P can lie. Give your answer in terms of π and surds.

Nearer to A than to B: P is on the A side of the perpendicular bisector of AB, a distance 2 cm from A
AP ≤ 4 cm: P is inside the quarter circle of radius 4 cm centred at A

The bisector meets the arc where $\cos\theta = \frac{2}{4}$, so θ = 60° at A. The region is a right-angled triangle plus a sector of angle 30°

Triangle: base 2 cm, height 4 sin 60° = $2\sqrt{3}$ cm, area = $\frac{1}{2} \times 2 \times 2\sqrt{3} = 2\sqrt{3}$

Sector of angle 30° : $\frac{30}{360} \times \pi \times 4^2 = \frac{4}{3}\pi$

Area = $2\sqrt{3} + \frac{4}{3}\pi$ cm^2

Answer: $(2\sqrt{3} + \frac{4}{3}\pi)$ cm^2

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10 Circle and line intersections

Solve the simultaneous equations

$x^2 + y^2 = 65$

$y = -x + 11$

Substitute into the first equation: $x^2 + (-x + 11)^2 = 65$

Expand: $2x^2 - 22x + 56 = 0$

Divide by 2: $x^2 - 11x + 28 = 0$

Factorise: $(x - 4)(x - 7) = 0$, so $x = 4$ or $x = 7$

$y = -x + 11$ gives $y = 7$ and $y = 4$

Answer: $x = 4, y = 7$ and $x = 7, y = 4$