Day 233 **S Tier 10-a-day: Model Solutions**

1 Ordering numbers

Write these numbers in order of size, starting with the smallest.

0.73, $\frac{14}{17}$, 42%, 0.84

Convert each to a decimal: 0.73 = 0.73, $\frac{14}{17}$ = 0.824, 42% = 0.42, 0.84 = 0.84

In order: 42%, 0.73, $\frac{14}{17}$, 0.84

Answer: 42%, 0.73, $\frac{14}{17}$, 0.84

2 Arc length and sector area

A sector of a circle has radius 9 cm.

The length of the arc of the sector is $\frac{3}{2}\pi$ cm.

(a) Work out the angle of the sector.

(b) Work out the area of the sector. Give your answer to (b) in terms of π.

(a) Arc length = $\frac{\theta}{360} \times 2\pi \times 9$, so $\frac{\theta}{360} \times 18 = \frac{3}{2}$

$\theta = \frac{3}{2} \times \frac{360}{18} = 30°$

(b) Area = $\frac{30}{360} \times \pi \times 9^2 = \frac{27}{4}\pi$ cm^2

Answer: (a) 30° (b) $\frac{27}{4}\pi$ cm^2

3 Iteration: rearranging equations

(a) Show that the equation $x^3 - 8x - 7 = 0$ has a solution between x = 3 and x = 4.

(b) Show that the equation can be rearranged to give $x = \sqrt[3]{8x + 7}$

(a) Let $f(x) = x^3 - 8x - 7$. $f(3) = 3^3 - 8 \times 3 - 7 = -4$ and $f(4) = 4^3 - 8 \times 4 - 7 = 25$

f(3) is negative and f(4) is positive. There is a change of sign, so there is a solution between 3 and 4

(b) Rearrange $x^3 - 8x - 7 = 0$: $x^3 = 8x + 7$

Take the cube root of both sides: $x = \sqrt[3]{8x + 7}$

Answer: Shown

4 Prime factors and indices

Write $9^2 \times 8^4$ as a product of powers of prime numbers.

$9 = 3^2$, $8 = 2^3$

$9^2 = 3^4$

$8^4 = 2^{12}$

Add the powers of each prime: $2^{12} \times 3^4$

Answer: $2^{12} \times 3^4$

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5 Similar triangles with algebra

In the diagram, DE is parallel to BC.
AD = x cm, DB = (x + 6) cm, DE = 3 cm and BC = 7 cm.
(a) Find the value of x.
(b) Work out the length of AB.
The diagram is not drawn accurately.

(a) Triangles ADE and ABC are similar, so $\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{x}{2x + 6} = \frac{3}{7}$
$7x = 3(2x + 6)$
$7x = 6x + 18$
$x = 18$
(b) $AB = 2x + 6 = 42$ cm
Answer: (a) x = 18 (b) 42 cm

6 Sample space and conditional probability

A fair 6-sided dice and a spinner numbered 1 to 5 are each thrown once. The first score and the second score are recorded.
(a) Work out the probability that the sum is greater than 5.
(b) Given that at least one number is 3, work out the probability that the sum is greater than 5.

(a) There are 6 × 5 = 30 equally likely outcomes, and 20 of them satisfy: the sum is greater than 5
Probability = $\frac{2}{3}$

(b) 10 outcomes satisfy: at least one number is 3. Of these, 6 also satisfy: the sum is greater than 5
Probability = $\frac{3}{5}$

Answer: (a) $\frac{2}{3}$ (b) $\frac{3}{5}$

7 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve. It crosses the axes at the points shown.
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The roots are $x = -1$ and $x = 6$, so $y = a(x + 1)(x - 6)$
When $x = 0$, $y = -18$: $a \times 1 \times (-6) = -18$, so $a = 3$
$y = 3(x + 1)(x - 6)$
$y = 3x^2 - 15x - 18$
Answer: $y = 3x^2 - 15x - 18$

8 Area of a triangle using sine

In triangle PQR, PQ = 15 cm and angle PQR = 60°.
The area of the triangle is $15\sqrt{3}$ cm^2.
Work out the length of QR.
Do not use a calculator.

Area = $\frac{1}{2} \times PQ \times QR \times \sin Q$

$15\sqrt{3} = \frac{1}{2} \times 15 \times QR \times \sin 60°$

$\sin 60° = \frac{\sqrt{3}}{2}$, so $15\sqrt{3} = \frac{15\sqrt{3}}{4} \times QR$

$QR = 15\sqrt{3} \div \frac{15\sqrt{3}}{4} = 4$ cm

Answer: 4 cm

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9 Ratio and algebra

In a coach, the ratio of the number of adults to the number of children is 8 : 7.
Then 8 more adults are added and 2 more children are added.
The ratio is now 4 : 3.
How many children were there at the start?

Let the numbers at the start be 8k and 7k
(8k + 8) : (7k + 2) = 4 : 3, so 3(8k + 8) = 4(7k + 2)
24k + 24 = 28k + 8
−4k = −16, so k = 4
Number of children at the start: 7 × 4 = 28
Answer: 28

10 Tangency conditions

The line $y = kx - 6$ is a tangent to the curve $y = x^2 - 2$.
Find the possible values of k.
Give any surds in their simplest form.

Equate: $x^2 - 2 = kx - 6$
Rearrange: $x^2 - kx + 4 = 0$
One point of contact, so $b^2 - 4ac = 0$: $k^2 - 4 \times 4 = 0$
$k^2 = 16$
$k = \pm\sqrt{16} = \pm 4$
Answer: k = ±4