Day 232 **S Tier 10-a-day: Model Solutions**

1 Ordering numbers

Write these numbers in order of size, starting with the smallest.

69%, $\frac{13}{18}$, 0.71, $\frac{1}{3}$

Convert each to a decimal: 69% = 0.69, $\frac{13}{18}$ = 0.722, 0.71 = 0.71, $\frac{1}{3}$ = 0.333

In order: $\frac{1}{3}$, 69%, 0.71, $\frac{13}{18}$

Answer: $\frac{1}{3}$, 69%, 0.71, $\frac{13}{18}$

2 Iteration

Use the iteration formula $x_{n+1} = \sqrt[3]{2x_n + 10}$ with $x_0 = 1$ (Calculator)
to work out x_1, x_2 and x_3.
Give each answer correct to 3 decimal places.

$x_1 = \sqrt[3]{2 \times 1 + 10} = 2.28943...$

$x_2 = \sqrt[3]{2 \times x_1 + 10} = 2.44291...$

$x_3 = \sqrt[3]{2 \times x_2 + 10} = 2.45994...$

Rounded: $x_1 = 2.289$, $x_2 = 2.443$, $x_3 = 2.460$

Answer: $x_1 = 2.289$, $x_2 = 2.443$, $x_3 = 2.460$

3 Exchange rates

Leila changes £700 into euros at £1 = €1.3.
Leila spends €340 and then changes the rest back into pounds at £1 = €1.5.
How many pounds does Leila get back?

£700 × 1.3 = €910
910 − 340 = €570
570 ÷ 1.5 = £380

Answer: £380

4 Equations in disguise

Solve $x - 13\sqrt{x} + 42 = 0$

Let $u = \sqrt{x}$: $u^2 - 13u + 42 = 0$
Factorise: $(u - 6)(u - 7) = 0$, so $u = 6$ or $u = 7$
$\sqrt{x} = 6$ or $\sqrt{x} = 7$
$x = 36$ or $x = 49$

Answer: $x = 36$ or $x = 49$

5 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the length OM.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) AM = 20 ÷ 2 = 10 cm
$OM^2 = OA^2 - AM^2 = 26^2 - 10^2 = 576$
OM = 24 cm

(b) sin(angle AOM) = $\frac{10}{26}$

Angle AOM = 22.62°, so angle AOB = 2 × 22.62 = 45.2°

Answer: (a) 24 cm (b) 45.2°

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6 Standard form problems

A atom has a diameter of 2×10^{-8} metres.
Atoms are placed in a straight line, touching, to make a length of 5.6×10^{-3} metres.
How many atoms are in the line?
Give your answer in standard form.

Number = total length ÷ diameter
$(5.6 \times 10^{-3}) \div (2 \times 10^{-8})$
$5.6 \div 2 = 2.8$ and $10^{-3} \div 10^{-8} = 10^{5}$
$= 2.8 \times 10^{5}$
Answer: 2.8×10^{5}

7 Ratio and algebra

In a fruit box, the ratio of the number of apples to the number of pears is 3 : 4.
Then 19 more apples are added and 8 more pears are added.
The ratio is now 1 : 1.
How many apples were there at the start?

Let the numbers at the start be 3k and 4k
$(3k + 19) : (4k + 8) = 1 : 1$, so $1(3k + 19) = 1(4k + 8)$
$3k + 19 = 4k + 8$
$-1k = -11$, so $k = 11$
Number of apples at the start: $3 \times 11 = 33$
Answer: 33

8 Volume and surface area of composite solids

A solid cylinder of radius 3 cm and height 13 cm has a hemispherical hollow of the same radius cut out of the top.
Work out the volume of the solid that is left. Give your answer in terms of π.

Cylinder: $\pi \times 3^2 \times 13 = 117\pi$
Hemisphere: $\frac{2}{3} \times \pi \times 3^3 = 18\pi$
Volume left = $117\pi - 18\pi = 99\pi$ cm^3
Answer: 99π cm^3

9 Percentages with an unknown rate

The price of a television is increased by x%.
The new price is then decreased by x%.
The final price is 25% less than the original price.
Work out the value of x.

Let the original price be 1. Multipliers: $(1 + \frac{x}{100})$ and $(1 - \frac{x}{100})$
$(1 + \frac{x}{100})(1 - \frac{x}{100}) = 1 - \frac{x^2}{10000}$
Final price is 25% less, so the multiplier is 0.75: $1 - \frac{x^2}{10000} = 0.75$
$\frac{x^2}{10000} = 0.25$, so $x^2 = 2500$ and $x = 50$
Answer: x = 50

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10 Recasting solids

A solid metal sphere of radius 6 cm is melted down and recast into a solid cylinder of radius 8 cm.
Work out the height of the cylinder.

The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

Volume of sphere = $\frac{4}{3} \times \pi \times 6^3 = 288\pi$

Volume of cylinder = $\pi \times 8^2 \times h = 64\pi h$

$64h = 288$

$h = \frac{9}{2}$ cm

Answer: $\frac{9}{2}$ cm