Day 231 **S Tier 10-a-day: Model Solutions**

1 Error intervals

The distance to a village is 3030 km, correct to 3 significant figures.
Write down the error interval for the distance to a village, using d.

The value has been rounded to the nearest 10
Half of this is 5, so go 5 below and 5 above
3030 − 5 = 3025 and 3030 + 5 = 3035
3025 ≤ d < 3035
Answer: 3025 ≤ d < 3035

2 Factorising quadratics

Factorise $6x^2 - 25x + 25$

6 × 25 = 150. Two numbers that multiply to 150 and add to −25 are −10 and −15
Split the middle term: $6x^2 - 10x - 15x + 25$
Factorise in pairs: $2x(3x - 5) - 5(3x - 5)$
$= (2x - 5)(3x - 5)$
Answer: (2x − 5)(3x − 5)

3 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle OAB = 69°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) OA = OB (radii), so triangle OAB is isosceles and angle OBA = 69°
Angle AOB = 180 − 2 × 69 = 42°
(b) The angle at the circumference is half the angle at the centre: 42 ÷ 2 = 21°
Answer: (a) 42° (b) 21°

4 Weighted percentages

The ratio of juniors to seniors in a school is 1 : 4.
85% of the juniors and 80% of the seniors play a sport.
What percentage of all of these students play a sport?

Imagine 100 students in total: 20 juniors and 80 seniors
85% of 20 = 17 and 80% of 80 = 64
Together: 81 out of 100
81 ÷ 100 × 100 = 81%
Answer: 81%

5 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 9^2 + 8^2 = 145$
$AG^2 = AC^2 + CG^2 = 145 + 12^2 = 289$
AG = 17 cm
(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{12}{\sqrt{145}}$
angle GAC = 44.9° (1 d.p.)
Answer: (a) 17 cm (b) 44.9°

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6 Composite functions

f(x) = 3x + 6 and g(x) = x^2 + 8

(a) Find gf(x). Give your answer in the form $ax^2 + bx + c$.

(b) Solve gf(x) = 17

(a) gf(x) = g(3x + 6) = $(3x + 6)^2$ + 8

= $9x^2$ + 36x + 44

(b) $(3x + 6)^2 = 9$, so 3x + 6 = ±3

3x + 6 = 3 or 3x + 6 = −3

x = −3 or x = −1

Answer: (a) $9x^2$ + 36x + 44 (b) x = −3 or x = −1

7 Probability and quadratic equations

A bag contains some counters. 4 of them are green and the rest are white.

Two counters are taken at random without replacement.

The probability that both are green is $\frac{6}{55}$.

Work out how many counters are white.

Let there be n counters altogether

P(both green) = $\frac{4}{n} \times \frac{3}{n-1} = \frac{12}{n(n-1)}$

$\frac{12}{n(n-1)} = \frac{6}{55}$, so $n(n-1) = 110$, giving $n^2 - n - 110 = 0$

(n − 11)(n + 10) = 0, so n = 11 (n cannot be negative)

Number of counters that are white = 11 − 4 = 7

Answer: 7

8 Transformations of graphs

The graph of y = f(x) has a minimum point at (−4, 2).

Write down the coordinates of the turning point of the graph of

(a) y = f(x) − 1

(b) y = f(x + 1) − 5

(a) Start with the turning point (−4, 2)

Apply the transformation to the point: (−4, 1)

(b) f(x + 1) changes the x-coordinate to −5, and f(x) − 5 changes the y-coordinate to −3

Turning point: (−5, −3)

Answer: (a) (−4, 1) (b) (−5, −3)

9 Bounds and accuracy

a = 2.35 and b = 0.89, both correct to 3 significant figures.

Work out a × b.

Write down your answer to a suitable degree of accuracy.

You must show your working and give a reason for your answer. (Calculator)

2.345 ≤ a < 2.355; 0.8895 ≤ b < 0.8905

Lower bound = 2.345 × 0.8895 = 2.0858775

Upper bound = 2.355 × 0.8905 = 2.0971275

Both bounds round to 2.1 to 2 significant figures, but not to 3 significant figures

Answer: 2.1 (2 s.f.)

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10 Tangency conditions

The line $y = 4x + k$ is a tangent to the curve $y = 3x^2 - x + 1$.
Find the value of k.

Equate: $3x^2 - x + 1 = 4x + k$

Rearrange: $3x^2 - 5x + 1 - k = 0$

A tangent touches at one point, so $b^2 - 4ac = 0$: $25 - 4 \times 3 \times (1 - k) = 0$

$12k + 13 = 0$

$k = -\frac{13}{12}$

Answer: $k = -\frac{13}{12}$