**Day 230** **S Tier 10-a-day: Model Solutions**

## 1 Angles in polygons

The interior angles of a pentagon are
$x°$, $(x - 5)°$, $(x + 10)°$, $(x - 20)°$, $(x + 30)°$
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = $(5 - 2) \times 180 = 540°$
$5x + 15 = 540$, so $5x = 525$
$x = 105$
(b) The angles are 105°, 100°, 115°, 85°, 135°
The largest is 135°
**Answer: (a) x = 105 (b) 135°**

## 2 Standard form

Work out $(4.32 \times 10^4) \div (1.6 \times 10^{-1})$
Give your answer in standard form.

$4.32 \div 1.6 = 2.7$
$10^4 \div 10^{-1} = 10^5$
$= 2.7 \times 10^5$
**Answer: $2.7 \times 10^5$**

## 3 Completing the square

(a) Write $x^2 + 2x + 7$ in the form $(x + p)^2 + q$
(b) Write down the coordinates of the turning point of the curve $y = x^2 + 2x + 7$

(a) Half of 2 is 1, so $x^2 + 2x = (x + 1)^2 - 1$
$x^2 + 2x + 7 = (x + 1)^2 - 1 + 7$
$= (x + 1)^2 + 6$
(b) The minimum is when $x + 1 = 0$, so $x = -1$ and $y = 6$
**Answer: (a) $(x + 1)^2 + 6$ (b) $(-1, 6)$**

## 4 Sine rule

The diagram shows triangle ABC. (Calculator)
Angle BAC = 51° and angle ABC is acute.
Work out the size of angle ABC, marked θ. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{\sin \theta}{AC} = \frac{\sin A}{BC}$
$\frac{\sin \theta}{8} = \frac{\sin 51°}{9}$
$\sin \theta = \frac{8 \times \sin 51°}{9} = 0.69080$
$\theta = \sin^{-1}(0.69080) = 43.7°$ (1 d.p.)
**Answer: $\theta = 43.7°$**

## 5 Speed-time graphs

The graph shows the speed of a train during 22 seconds.
(a) Work out the acceleration during the first 8 seconds.
(b) Work out the average speed over the whole 22 seconds.

(a) Acceleration = change in speed ÷ time = $(14 - 6) \div 8 = 1$ m/s$^2$
(b) Distance = area under the graph
Trapezium: $\frac{6 + 14}{2} \times 8 = 80$; rectangle: $14 \times 6 = 84$; triangle: $\frac{1}{2} \times 8 \times 14 = 56$
Total = 220 m, so average speed = $220 \div 22 = 10$ m/s
**Answer: (a) 1 m/s$^2$ (b) 10 m/s**

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## 6 Area and quadratic equations

The diagram shows a rectangle of length (2x + 1) cm, width (2x − 3) cm and area 357 cm$^2$.
(a) Show that $4x^2 - 4x - 360 = 0$.
(b) Work out the length of the diagonal of the rectangle. Give your answer correct to 3 significant figures.

(a) $(2x + 1)(2x - 3) = 357$
Expand: $4x^2 - 4x - 3 = 357$, so $4x^2 - 4x - 360 = 0$
(b) Factorise: $4(x - 10)(x + 9) = 0$
$x = 10$ ($x = -9$ is not possible as a length would be negative)
Length = 21 cm and width = 17 cm
Diagonal = $\sqrt{21^2 + 17^2} = \sqrt{730}$ = 27.0 cm (3 s.f.)
**Answer: (b) 27.0 cm**

## 7 Transformations of graphs

The point P(6, −3) lies on the graph of y = f(x).
Write down the coordinates of the image of P on the graph of
(a) y = −f(x) (b) y = $\frac{1}{2}$f(x)

(a) y = −f(x) is a reflection in the x-axis, so the y coordinate changes sign: (6, 3)
(b) y = $\frac{1}{2}$f(x) is a vertical stretch with scale factor $\frac{1}{2}$, so the y coordinate is halved: (6, $-\frac{3}{2}$)

**Answer: (a) (6, 3) (b) (6, $-\frac{3}{2}$)**

## 8 Recurring decimals

Show that $0.7\dot{6}\dot{8} = \frac{761}{990}$

Let x = 0.7686...
10x = 7.6868...
1000x = 768.6868...
Subtract: 990x = 761
$x = \frac{761}{990}$

**Answer: x = $\frac{761}{990}$**

## 9 Tree diagrams and reverse probability

The probability that it rains on a day is 0.3.
If it rains, the probability that Ali is late for school is 0.3.
If it does not rain, the probability that Ali is late is 0.1.
(a) Work out the probability that Ali is late.
(b) Ali is late. Work out the probability that it rained.

(a) P(late) = 0.3 × 0.3 + 0.7 × 0.1 = 0.09 + 0.07 = 0.16
(b) P(rain and late) = 0.09
P(rain | late) = 0.09 ÷ 0.16 = $\frac{9}{16}$

**Answer: (a) 0.16 (b) $\frac{9}{16}$**

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## 10 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 + 7x + 10}{x - 3} \div \frac{x + 2}{x^2 + x - 12}$

Factorise: $\frac{(x + 5)(x + 2)}{(x - 3)} \div \frac{(x + 2)}{(x - 3)(x + 4)}$

Turn the second fraction upside down and multiply: $\frac{(x + 5)(x + 2)}{(x - 3)} \times \frac{(x - 3)(x + 4)}{(x + 2)}$

Cancel $(x + 2)$ and $(x - 3)$
$= (x + 4)(x + 5)$
**Answer: $(x + 4)(x + 5)$**