Day 23 **S Tier 10-a-day: Model Solutions**

1 Surds with unknowns

Given that $(a + \sqrt{10})(b + \sqrt{10}) = 40 + 11\sqrt{10}$,
where a and b are positive integers with a < b, find the values of a and b.

$(a + \sqrt{10})(b + \sqrt{10}) = ab + (a + b)\sqrt{10} + 10$
Compare the $\sqrt{10}$ terms: a + b = 11
Compare the integer terms: ab + 10 = 40, so ab = 30
Two numbers with sum 11 and product 30 are 5 and 6
Answer: a = 5, b = 6

2 Mean of a changing set

The mean of 5 numbers is 155.
One number is removed and the mean of the remaining 4 numbers is 189.75.
Work out the number that was removed.

Total of the 5 numbers = 5 × 155 = 775
Total of the 4 remaining = 4 × 189.75 = 759
Number removed = 775 − 759 = 16
Answer: 16

3 Time zones

A flight leaves London at 08:20 (London time).
It takes 11 hours 45 minutes.
When it is 12:00 in London it is 21:00 in Sydney.
At what time, in Sydney time, does the flight land? Say if it is on a different day.

Arrival in London time = 08:20 + 11 h 45 min = 20:05
Sydney is 9 hours ahead of London, so add 9 hours
05:05 the next day
Answer: 05:05 the next day

4 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 68°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 68°
Angle COB is an exterior angle of triangle OAC: 68 + 68 = 136°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 136°

5 Quadratic sequences

Here are the first five terms of a quadratic sequence.
−2, −2, 0, 4, 10
Find an expression for the nth term.

First differences: 0, 2, 4, 6
Second differences: 2, 2, 2, so a = 2 ÷ 2 = 1
Subtract n^2 from each term: −3, −6, −9, −12, −15
The remaining linear sequence has nth term −3n
nth term = $n^2 - 3n$
Answer: $n^2 - 3n$

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6 Compound interest

Evan invests £9,500. The rates of interest for the next 3 years are 4%, 2% and 2%.
Compound interest is paid. Work out the value of the investment after 3 years.
Give your answer to the nearest penny. (Calculator)

Multipliers: 1.04, 1.02 and 1.02
9500 × 1.04 × 1.02 × 1.02 = 10279.15
To the nearest penny: £10,279.15
Answer: £10,279.15

7 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 2^2 + 11^2 = 125$
$AG^2 = AC^2 + CG^2 = 125 + 10^2 = 225$
AG = 15 cm

(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{10}{\sqrt{125}}$

angle GAC = 41.8° (1 d.p.)
Answer: (a) 15 cm (b) 41.8°

8 Finding a percentage rate

Lena invests £17,500 in an account paying compound interest.
The interest rate is the same percentage each year.
After 3 years the account holds £30,240.
Work out the annual percentage interest rate.

Multiplier for 3 years = £30,240 ÷ £17,500 = 1.728
Yearly multiplier = the cube root of 1.728 = 1.2
20% increase each year
Answer: 20%

9 Equations with algebraic fractions

Solve $\frac{3}{x+4} - \frac{2}{x-3} = \frac{3}{2}$

Multiply every term by $(x + 4)(x - 3)$: $3(x - 3) - 2(x + 4) = \frac{3}{2} \times (x + 4)(x - 3)$

Multiply by 2 to clear the fraction: $6(x - 3) - 4(x + 4) = 3(x + 4)(x - 3)$

Expand and rearrange so that one side is 0: $3x^2 + x - 2 = 0$

Factorise: $(x + 1)(3x - 2) = 0$

$x = -1$ or $x = \frac{2}{3}$

Answer: $x = -1$ or $x = \frac{2}{3}$

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10 Ratio and money transferred

Sofia and Carlos share some money in the ratio 9 : 1.
Sofia gives Carlos £51.
The ratio of their money is now 1 : 2.
How much money do they have altogether?

Let the amounts be 9k and 1k, so the total is 10k

Sofia now has $9k - 51$, which is $\frac{1}{3}$ of the total

$3(9k - 51) = 1 \times 10k$
$27k - 153 = 10k$, so $17k = 153$ and $k = 9$
Total = $10 \times 9 = £90$
Answer: £90