Day 229 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

The interior angles of a pentagon are
$x°$, $(x + 25)°$, $(x + 10)°$, $(x + 15)°$, $(2x - 20)°$
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = $(5 - 2) \times 180 = 540°$
$6x + 30 = 540$, so $6x = 510$
$x = 85$
(b) The angles are 85°, 110°, 95°, 100°, 150°
The largest is 150°
Answer: (a) x = 85 (b) 150°

2 Multiplying and dividing surds

Simplify fully $5\sqrt{20} \times 4\sqrt{21}$

$5\sqrt{20} \times 4\sqrt{21} = 20\sqrt{420}$
$\sqrt{420} = 2\sqrt{105}$, so $20 \times 2\sqrt{105} = 40\sqrt{105}$
Answer: $40\sqrt{105}$

3 Comparing costs

FitLife has a joining fee of £1 plus £3 per month.
GymNow has a joining fee of £29 plus £1 per month.
(a) Work out the number of months for which the two costs are the same.
(b) Which is cheaper for 20 months, and by how much?

(a) Let n be the number of months. Working in pounds, the costs are equal when $1 + 3n = 29 + n$
$2n = 28$, so $n = 14$
(b) FitLife: £1 + £3 × 20 = £61
GymNow: £29 + £1 × 20 = £49
GymNow is cheaper by £12
Answer: (a) 14 (b) GymNow, by £12

4 Exchange rates

Grace changes £300 into dollars at £1 = $1.3.
Grace spends $280 and then changes the rest back into pounds at £1 = $1.6.
How many pounds does Grace get back?

£300 × 1.3 = $390
390 − 280 = $110
110 ÷ 1.6 = £68.75
Answer: £68.75

5 Fractional indices

Given that $x > 0$, solve $x^{\frac{1}{3}} = 4$

Raise both sides to the power 3
$x = (4)^3$
$x = 64$
Answer: x = 64

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6 Combined transformations

The point P(−3, −1) is reflected in the y-axis.
The image is then reflected in the line y = x.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (3, −1)
After the second transformation: P'' = (−1, 3)
(b) Follow a general point (x, y) through both transformations: (x, y) → (−x, y) → (y, −x)
The single mapping is (x, y) → (y, −x)
This is a rotation 90° clockwise about the origin
Answer: (a) (−1, 3) (b) Rotation 90° clockwise about the origin

7 Inverse proportion

P is inversely proportional to t.
When t = 3, P = 2.
Find the value of P when t = 2.

$P = \frac{k}{t}$

$2 = \frac{k}{3}$, so $k = 2 \times 3 = 6$

$P = \frac{6}{2} = 3$

Answer: P = 3

8 Expanding brackets with surds

Expand and simplify $(1 + 3\sqrt{6})(2 + \sqrt{6})$
Give your answer in the form $p + q\sqrt{6}$

$(1 + 3\sqrt{6})(2 + \sqrt{6}) = 2 + \sqrt{6} + 6\sqrt{6} + 18$
$= 20 + 7\sqrt{6}$
Answer: $20 + 7\sqrt{6}$

9 Vector proof: collinear points

OA = **a** and OB = **b**.
OP = 5**a**, OQ = 14**a** + 3**b** and OR = 20**a** + 5**b**.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = 9**a** + 3**b**
QR = OR − OQ = 6**a** + 2**b**
$QR = \frac{2}{3}$ PQ, so PQ and QR are parallel, and they share the point Q
So P, Q and R lie on a straight line
(b) PQ : QR = 3 : 2
Answer: (b) 3 : 2

10 Tangent to a circle: proof

Show that the line y = −x + 14 is a tangent to the circle $x^2 + y^2 = 98$.
Find the coordinates of the point where the line touches the circle.

Substitute y = −x + 14 into $x^2 + y^2 = 98$: $x^2 + (-x + 14)^2 = 98$
Expand: $2x^2 - 28x + 98 = 0$
$b^2 - 4ac = (-28)^2 - 4 \times 2 \times 98 = 784 - 784 = 0$
$b^2 - 4ac = 0$, so there is exactly one point of intersection: the line is a tangent
$x = \frac{-(-28)}{2 \times 2} = 7$, and y = −7 + 14 = 7. The point is (7, 7)
Answer: (7, 7)