Day 228 **S Tier 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 20:40 (London time).
It takes 13 hours 40 minutes.
When it is 12:00 in London it is 07:00 in New York.
At what time, in New York time, does the flight land? Say if it is on a different day.

Arrival in London time = 20:40 + 13 h 40 min = 10:20 the next day
New York is 5 hours behind London, so subtract 5 hours
05:20 the next day
Answer: 05:20 the next day

2 Arithmetic sequences

The 2nd term of an arithmetic sequence is 14.
The 9th term of the sequence is 35.
(a) Find an expression for the nth term of the sequence.
(b) Is 68 a term of the sequence? You must show your working.

(a) Common difference = $(35 - 14) \div (9 - 2) = 3$
nth term = $3n + c$. For $n = 2$: $3 \times 2 + c = 14$, so $c = 8$
nth term = $3n + 8$
(b) Solve $3n + 8 = 68$: $3n = 60$, so $n = 20$
n is a positive whole number, so 68 is the 20th term
Answer: (a) $3n + 8$ (b) Yes, it is the 20th term

3 Fractional indices

Given that $x > 0$, solve $x^{\frac{5}{3}} = 1024$

Raise both sides to the power $\frac{3}{5}$

$x = (1024)^{\frac{3}{5}}$
$x = 64$
Answer: $x = 64$

4 Combined transformations

The point P(5, −4) is reflected in the x-axis.
The image is then reflected in the line $y = x$.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (5, 4)
After the second transformation: P'' = (4, 5)
(b) Follow a general point (x, y) through both transformations: $(x, y) \rightarrow (x, -y) \rightarrow (-y, x)$
The single mapping is $(x, y) \rightarrow (-y, x)$
This is a rotation 90° anticlockwise about the origin
Answer: (a) (4, 5) (b) Rotation 90° anticlockwise about the origin

5 Iteration

Use the iteration formula $x_{n+1} = \sqrt[3]{4x_n + 2}$ with $x_0 = 4$ (Calculator)
to work out x_1, x_2 and x_3.
Give each answer correct to 3 decimal places.

$x_1 = \sqrt[3]{4 \times 4 + 2} = 2.62074...$
$x_2 = \sqrt[3]{4 \times x_1 + 2} = 2.31974...$
$x_3 = \sqrt[3]{4 \times x_2 + 2} = 2.24262...$
Rounded: $x_1 = 2.621$, $x_2 = 2.320$, $x_3 = 2.243$
Answer: $x_1 = 2.621$, $x_2 = 2.320$, $x_3 = 2.243$

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6 Exponential growth and decay

A population of members of a website starts at 800 and grows by 50% each year.
After how many whole years will the population first be at least double its starting value? (Calculator)

Multiplier = 1.5, double is 1,600

After 1 years: $800 \times 1.5^1 = 1200.0$

After 2 years: $800 \times 1.5^2 = 1800.0$

Answer: 2 years

7 Index equations

Solve $8^{2x+3} = 16^3$

Write both sides as powers of 2: $8 = 2^3$ and $16 = 2^4$

Equate the powers: $6x + 9 = 12$

$6x = 3$, so $x = \frac{1}{2}$

Answer: $x = \frac{1}{2}$

8 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $4\mathbf{b}$, OQ = $-3\mathbf{a} + 5\mathbf{b}$ and OR = $-9\mathbf{a} + 7\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $-3\mathbf{a} + \mathbf{b}$

QR = OR − OQ = $-6\mathbf{a} + 2\mathbf{b}$

QR = 2 PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line

(b) PQ : QR = 1 : 2

Answer: (b) 1 : 2

9 Trigonometric graphs

The diagram shows the graph of $y = a \cos x° + c$ for $0 \leq x \leq 360$.
Some points on the graph are labelled.
Find the value of a and the value of c.

At x = 0, cos x° = 1, so a + c = 6

At x = 180, cos x° = −1, so −a + c = 2

Add: 2c = 8, so c = 4

Subtract: 2a = 4, so a = 2

Answer: a = 2, c = 4

10 Estimating the mean from a histogram table

The table gives information about the ages of some people.
Work out an estimate for the mean age.
Give your answer correct to 1 decimal place. (Calculator)

Frequency = frequency density × class width: 18, 16, 39, 64

Mid-points: 5, 20, 45, 80

Σfm = 18×5 + 16×20 + 39×45 + 64×80 = 7285, Σf = 137

Mean = 7285 ÷ 137 = 53.2

Answer: 53.2