Day 227 **S Tier 10-a-day: Model Solutions**

1 Equations with indices

Solve $x^{\frac{2}{3}} = 4$
$x > 0$

Write 4 as a power of 2: $4 = 2^2$
$x^{\frac{2}{3}} = 2^2$, so $x = (2^2)^{\frac{3}{2}}$
$x = 2^3 = 8$
Answer: x = 8

2 Distance-time graphs

The graph shows Priya's cycle ride to a café and back.
(a) Work out Priya's speed on the way back.
(b) Work out Priya's average speed for the whole ride, including the stop.

(a) Return: 36 km in 4.5 hours, so speed = 36 ÷ 4.5 = 8 km/h
(b) Total distance = 2 × 36 = 72 km, total time = 7.5 hours
Average speed = 72 ÷ 7.5 = 9.6 km/h
Answer: (a) 8 km/h (b) 9.6 km/h

3 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the length of the chord AB.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AM^2 = OA^2 - OM^2 = 10^2 - 8^2 = 36$
AM = 6 cm
AB = 2 × 6 = 12 cm

(b) sin(angle AOM) = $\frac{6}{10}$
Angle AOM = 36.87°, so angle AOB = 2 × 36.87 = 73.7°
Answer: (a) 12 cm (b) 73.7°

4 Rationalising the denominator

Rationalise the denominator of $\frac{24}{4\sqrt{7}}$
Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{7}$:
$\frac{24}{4\sqrt{7}} = \frac{24\sqrt{7}}{28}$
$= \frac{6\sqrt{7}}{7}$

Answer: $\frac{6\sqrt{7}}{7}$

5 Exact trigonometric values

Work out the exact value of $(\tan 60°)^2 + \sin 60°$
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$(\tan 60°)^2 + \sin 60° = (\sqrt{3})^2 + \frac{\sqrt{3}}{2}$

$= 3 + \frac{\sqrt{3}}{2}$

Answer: $3 + \frac{\sqrt{3}}{2}$

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6 Tree diagrams without replacement

A bag contains 2 orange cubes, 4 purple cubes and 1 pink cube.
Two cubes are taken at random without replacement.
Work out the probability that (a) they are different colours, (b) at least one is orange.

(a) P(different) = 1 − P(same) = $1 - \frac{1}{3} = \frac{2}{3}$

(b) P(at least one orange) = 1 − P(none) = $1 - \frac{5}{7} \times \frac{4}{6} = \frac{11}{21}$

Answer: (a) $\frac{2}{3}$ (b) $\frac{11}{21}$

7 Simultaneous equations

Solve the simultaneous equations
$3x - y = 25$
$6x + y = 38$

To match the x terms: (1) × 2 gives $6x - 2y = 50$
Subtract the equations to eliminate x: $-3y = 12$
$y = -4$
Substitute $y = -4$ into (1): $3x + 4 = 25$
$3x = 21$, so $x = 7$
Answer: x = 7, y = −4

8 Fractional indices

Given that $x > 0$, solve $x^{\frac{5}{4}} = 1024$

Raise both sides to the power $\frac{4}{5}$

$x = (1024)^{\frac{4}{5}}$
$x = 256$
Answer: x = 256

9 Tangent to a circle

A circle has equation $x^2 + y^2 = 169$.
The point P(5, 12) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $\frac{12}{5}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div \frac{12}{5} = -\frac{5}{12}$

$y - 12 = (-\frac{5}{12})(x - 5)$

Multiply out and rearrange: $5x + 12y = 169$
(b) On the x-axis $y = 0$, so $5x = 169$
$x = \frac{169}{5}$, so T is $(\frac{169}{5}, 0)$

Answer: (a) 5x + 12y = 169 (b) T = $(\frac{169}{5}, 0)$

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10 Bounds and accuracy

a = 33 and b = 94, both correct to 3 significant figures.
Work out a ÷ b.
Write down your answer to a suitable degree of accuracy.
You must show your working and give a reason for your answer. (Calculator)

$32.95 \leq a < 33.05$; $93.95 \leq b < 94.05$
Lower bound = 32.95 ÷ 94.05 = 0.35035...
Upper bound = 33.05 ÷ 93.95 = 0.35178...
Both bounds round to 0.35 to 2 significant figures, but not to 3 significant figures
Answer: 0.35 (2 s.f.)