Day 226 **S Tier 10-a-day: Model Solutions**

1 Cyclic quadrilaterals

A, B, C and D are points on a circle. Angle DAB = (x + 30)°, angle BCD = (5x + 6)° and angle ABC = (3x + 10)°.
(a) Work out the value of x.
(b) Work out the size of angle ADC.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
(x + 30) + (5x + 6) = 180
6x + 36 = 180, so 6x = 144
x = 24
(b) Angle ABC = 3 × 24 + 10 = 82°
Angle ADC = 180 − 82 = 98°
Answer: (a) x = 24 (b) 98°

2 Equations with fractions

Solve $\frac{x+1}{4} + \frac{x-5}{2} = 3$

Multiply every term by 4: (x + 1) + 2(x − 5) = 12
Expand: 3x − 9 = 12
3x = 21
x = 7
Answer: x = 7

3 Pressure

A metal crate in the shape of a cuboid weighs 570 N.
Its smallest face has area 1000 cm^2. Its largest face has area 2,000 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the metal crate can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 1000 cm^2 = 0.1 m^2 and 2,000 cm^2 = 0.2 m^2
(a) Greatest pressure is on the smallest face: 570 ÷ 0.1 = 5,700 N/m^2
(b) Least pressure is on the largest face: 570 ÷ 0.2 = 2,850 N/m^2
Answer: (a) 5,700 N/m^2 (b) 2,850 N/m^2

4 Percentages

S is 50% less than C.
C is 80% more than R.
Work out S as a percentage of R.

S = 0.5 × C and C = 1.8 × R
S = 0.5 × 1.8 × R = 0.9 × R
0.9 × 100 = 90%
Answer: 90%

5 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle APB = 50°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angles in quadrilateral OAPB: 360 − 90 − 90 − 50 = 130°
(b) The angle at the circumference is half the angle at the centre: 130 ÷ 2 = 65°
Answer: (a) 130° (b) 65°

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6 Percentage change in area and volume

The radius of a circle is decreased by 20%.
Work out the percentage change in the area of the circle.

Radius multiplier = 0.8

Area multiplier = $0.8^2 = 0.64$

$0.64 - 1 = -0.36$, so a 36% decrease

Answer: 36% decrease

7 Perpendicular bisector

A is the point $(-2, 0)$ and B is the point $(6, 2)$.
Find an equation of the perpendicular bisector of AB. Give your answer in the form $ax + by = c$, where a, b and c are integers.

Midpoint of AB = (2, 1)

Gradient of AB = $\frac{1}{4}$

Perpendicular gradient = -4

$y - 1 = (-4)(x - 2)$

Rearrange to integer coefficients: $4x + y = 9$

Answer: $4x + y = 9$

8 Direct proportion formulae

C is directly proportional to the cube of d.
When d = 2, C = 56.
(a) Find a formula for C in terms of d.
(b) Work out the value of C when d = 3.

(a) $C = kd^3$

$56 = k \times 8$, so $k = 56 \div 8 = 7$

$C = 7d^3$

(b) $C = 7 \times 27 = 189$

Answer: (a) $C = 7d^3$ (b) 189

9 Tangency conditions

The line $y = kx - 3$ is a tangent to the curve $y = x^2 + 1$.
Find the possible values of k.
Give any surds in their simplest form.

Equate: $x^2 + 1 = kx - 3$

Rearrange: $x^2 - kx + 4 = 0$

One point of contact, so $b^2 - 4ac = 0$: $k^2 - 4 \times 4 = 0$

$k^2 = 16$

$k = \pm\sqrt{16} = \pm 4$

Answer: $k = \pm 4$

10 Percentage change and proportion

The time for a pendulum to swing is directly proportional to the square root of the length of the pendulum.
The length of the pendulum is increased by 96%.
Work out the percentage change in the time for a pendulum to swing.

The length of the pendulum is multiplied by 1.96

The multiplier for the time for a pendulum to swing is $\sqrt{1.96} = 1.4$

1.4 is an increase of 40%

Answer: 40% increase