Day 225 **S Tier 10-a-day: Model Solutions**

1 Hollow cylinder and density

A metal pipe is a hollow cylinder of length 25 cm. (Calculator)
The outer radius is 10 cm and the inner radius is 9.5 cm.
The density of the metal is 2.7 g/cm^3.
Work out the mass of the pipe. Give your answer in kilograms correct to 3 significant figures.

Volume of metal = $\pi \times 10^2 \times 25 - \pi \times 9.5^2 \times 25 = \pi(100 - 90.25) \times 25$
$= 765.76$ cm^3
Mass = density × volume = 2.7 × 765.76 = 2067.6 g
= 2.07 kg (3 s.f.)
Answer: 2.07 kg

2 Rearranging when the subject appears twice

Make x the subject of $y = \frac{4x + 2}{2x + 4}$

Multiply by 2x + 4: $y(2x + 4) = 4x + 2$
Expand: $2xy + 4y = 4x + 2$
Collect the x terms: $2xy - 4x = 2 - 4y$
Factorise: $x(2y - 4) = -4y + 2$
$x = \frac{-4y + 2}{2y - 4}$
Answer: $x = \frac{-4y + 2}{2y - 4}$

3 Fractions

Work out $(1\frac{1}{3} - \frac{2}{3}) \div 2\frac{1}{2}$
Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions
$\frac{4}{3} - \frac{2}{3} = \frac{2}{3}$
$\frac{2}{3} \div \frac{5}{2} = \frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$
Answer: $\frac{4}{15}$

4 Comparing costs

FitLife has a joining fee of £5 plus £5.50 per month.
GymNow has a joining fee of £13 plus £3.50 per month.
(a) Work out the number of months for which the two costs are the same.
(b) Which is cheaper for 7 months, and by how much?

(a) Let n be the number of months. Working in pounds, the costs are equal when $5 + 5.50n = 13 + 3.50n$
$2n = 8$, so $n = 4$
(b) FitLife: £5 + £5.50 × 7 = £43.50
GymNow: £13 + £3.50 × 7 = £37.50
GymNow is cheaper by £6
Answer: (a) 4 (b) GymNow, by £6

5 Exchange rates

Grace changes £300 into euros at £1 = €1.18.
Grace spends €80 and then changes the rest back into pounds at £1 = €1.6.
How many pounds does Grace get back?

£300 × 1.18 = €354
354 − 80 = €274
274 ÷ 1.6 = £171.25
Answer: £171.25

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6 Iteration

Use the iteration formula $x_{n+1} = \sqrt{3x_n + 1}$ with $x_0 = 3$ (Calculator)
to work out x_1, x_2 and x_3.
Give each answer correct to 3 decimal places.

$x_1 = \sqrt{3 \times 3 + 1} = 3.16228...$

$x_2 = \sqrt{3 \times x_1 + 1} = 3.23834...$

$x_3 = \sqrt{3 \times x_2 + 1} = 3.27338...$

Rounded: $x_1 = 3.162$, $x_2 = 3.238$, $x_3 = 3.273$

Answer: $x_1 = 3.162$, $x_2 = 3.238$, $x_3 = 3.273$

7 Perpendicular bisector

A is the point (1, 0) and B is the point (−7, −4).
Find an equation of the perpendicular bisector of AB. Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = (−3, −2)

Gradient of AB = $\frac{1}{2}$

Perpendicular gradient = −2

y − (−2) = (−2)(x − (−3))

Rearrange to integer coefficients: 2x + y = −8

Answer: 2x + y = −8

8 3D problems: pyramids

VABCD is a pyramid on a square base. V is directly above the centre M. (Calculator)
(a) Work out AC as a simplified surd.
(b) Work out the angle between VA and the base, to 1 d.p.
The diagram is not drawn accurately.

(a) $AC^2 = 8^2 + 8^2 = 128$

$AC = \sqrt{128} = 8\sqrt{2}$ cm

(b) AM = AC ÷ 2 = $4\sqrt{2}$ cm. The angle is VAM.

$\tan(VAM) = \frac{VM}{AM} = \frac{2}{\sqrt{32}}$

angle VAM = 19.5° (1 d.p.)

Answer: (a) $8\sqrt{2}$ cm (b) 19.5°

9 Tree diagrams and reverse probability

The probability that it rains on a day is 0.3.
If it rains, the probability that Ali is late for school is 0.7.
If it does not rain, the probability that Ali is late is 0.1.
(a) Work out the probability that Ali is late.
(b) Ali is late. Work out the probability that it rained.

(a) P(late) = 0.3 × 0.7 + 0.7 × 0.1 = 0.21 + 0.07 = 0.28

(b) P(rain and late) = 0.21

P(rain | late) = 0.21 ÷ 0.28 = $\frac{3}{4}$

Answer: (a) 0.28 (b) $\frac{3}{4}$

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10 Profit and loss

A shop buys a printer for £140.
It adds a 20% mark-up to get the selling price.
In a sale the selling price is reduced by 30%.
Work out the percentage profit or loss on the original cost.

Marked price = £140 × 1.2 = £168
Sale price = £168 × 0.7 = £117.60
Loss = £22.40
Percentage = $\frac{22.4}{140}$ × 100 = 16%

Answer: 16% loss