Day 224 **S Tier 10-a-day: Model Solutions**

1 Capture and recapture

30 fish in a lake are caught, tagged and put back.
Later, a second sample of 120 fish is caught. 3 of these are tagged.
(a) Work out an estimate for the number of fish in a lake.
(b) State one assumption you have made.

(a) The proportion tagged in the second sample is 3 out of 120

This should match the proportion tagged in the whole population: $\frac{3}{120} = \frac{30}{N}$

N = 30 × 120 ÷ 3 = 1200

(b) No fish are born, die or leave between the samples, and the fish mix evenly

Answer: (a) 1200 (b) The population stays the same between samples

2 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve with a turning point at (−2, −6).
The curve crosses the y-axis at (0, 6).
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The turning point is (−2, −6), so $y = a(x + 2)^2 - 6$
When x = 0, y = 6: a × 4 − 6 = 6, so a = 3
$y = 3(x + 2)^2 - 6$
Expand: $y = 3x^2 + 12x + 6$

Answer: $y = 3x^2 + 12x + 6$

3 Fractions

Work out $(2\frac{1}{2} - \frac{1}{2}) \div 2\frac{1}{5}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions

$\frac{5}{2} - \frac{1}{2} = 2$

$2 \div \frac{11}{5} = 2 \times \frac{5}{11} = \frac{10}{11}$

Answer: $\frac{10}{11}$

4 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 4 cm, DB = 10 cm and the area of triangle ADE is 80 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 4 + 10 = 14 cm

Length scale factor = 14 ÷ 4 = $\frac{7}{2}$

Area scale factor = $(\frac{7}{2})^2 = \frac{49}{4}$

Area of triangle ABC = 80 × $\frac{49}{4}$ = 980 cm^2

Trapezium = 980 − 80 = 900 cm^2

Answer: 900 cm^2

5 Equation of a circle

A circle has centre C(−5, 3) and passes through the point P(3, 0).
Find an equation of the circle.

$r^2 = CP^2 = (3 - (-5))^2 + (0 - 3)^2$
$= 64 + 9 = 73$
$(x + 5)^2 + (y - 3)^2 = 73$

Answer: $(x + 5)^2 + (y - 3)^2 = 73$

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6 Mixtures and percentage profit

A shop mixes two types of tea.
Type A costs £8.00 per kg and type B costs £11.00 per kg.
They are mixed in the ratio A : B = 1 : 4.
The mixture is sold for £14.56 per kg.
Work out the percentage profit.

Cost of 5 kg of mixture = 1 × £8.00 + 4 × £11.00 = £52.00
Cost per kg = £52.00 ÷ 5 = £10.40
Profit per kg = £14.56 − £10.40 = £4.16
Percentage profit = 4.16 ÷ 10.40 × 100 = 40%
Answer: 40%

7 Compound interest

Nia puts £3,000 in a savings account.
Compound interest is paid at 4% in year 1, 5% in year 2 and 4% in year 3.
Work out the amount in the account at the end of year 3.
Give your answer to the nearest penny. (Calculator)

Multipliers: 1.04, 1.05, 1.04
£3,000 × 1.04 × 1.05 × 1.04
= £3,407.04
Answer: £3,407.04

8 Reverse percentage growth and decay

The value of a house increases by 20% each year.
After 2 years it is worth £7,200.
Work out its value 2 years ago. (Calculator)

Yearly multiplier = 1.2
Original × 1.2^2 = £7,200
Original = £7,200 ÷ 1.2^2 = £5,000
Answer: £5,000

9 Functions that are their own inverse

$f(x) = \frac{3x + 2}{x - 3}$ where $x \neq 3$
(a) Show that ff(x) = x.
(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by (x − 3)
$ff(x) = \frac{3(3x + 2) + 2(x - 3)}{(3x + 2) - 3(x - 3)}$
Top = 9x + 6 + 2x − 6 = 11x
Bottom = 3x + 2 − 3x + 9 = 11
$ff(x) = \frac{11x}{11} = x$

(b) ff(x) = x, so f is its own inverse: $f^{-1}(x) = \frac{3x + 2}{x - 3}$

Answer: (b) $\mathbf{f^{-1}(x) = \frac{3x + 2}{x - 3}}$

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10 Transformations of graphs

The graph of y = f(x) has a minimum point at (2, −4).
Write down the coordinates of the turning point of the graph of
(a) y = f(x/2)
(b) y = $\frac{1}{2}$f(x + 4)

(a) Start with the turning point (2, −4)
Apply the transformation to the point: (4, −4)

(b) f(x + 4) changes the x-coordinate to −2, and $\frac{1}{2}$f(x) changes the y-coordinate to −2

Turning point: (−2, −2)
Answer: (a) (4, −4) (b) (−2, −2)