Day 223 **S Tier 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 07:45 (London time).
It takes 7 hours 35 minutes.
When it is 12:00 in London it is 04:00 in Los Angeles.
At what time, in Los Angeles time, does the flight land? Say if it is on a different day.

Arrival in London time = 07:45 + 7 h 35 min = 15:20
Los Angeles is 8 hours behind London, so subtract 8 hours
07:20
Answer: 07:20 the same day

2 Quadratic graphs

The curve $y = x^2 - 10x + 16$ crosses the x-axis at the points A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the minimum point of the curve.

(a) Put $y = 0$: $x^2 - 10x + 16 = 0$, so $(x - 2)(x - 8) = 0$
$x - 2 = 0$ or $x - 8 = 0$, so $x = 2$ or $x = 8$
(b) The turning point is halfway between A and B: $x = (2 + 8) \div 2 = 5$
When $x = 5$: $y = 5^2 - 10 \times 5 + 16 = -9$
Answer: (a) A(2, 0), B(8, 0) (b) (5, −9)

3 Recurring decimals

Write $0.79\dot{2}$ as a fraction in its simplest form.

Let x = 0.7922...
100x = 79.2222...
1000x = 792.2222...
Subtract: 900x = 713
$x = \frac{713}{900}$
Answer: $\mathbf{\frac{713}{900}}$

4 Using a line of best fit

The scatter graph shows temperature (x) and ice creams sold (y).
The equation of the line of best fit is $y = 2.5x + 9$.
(a) Estimate y when $x = 9$.
(b) Explain why an estimate of y when $x = 20$ may be unreliable.

(a) $y = 2.5 \times 9 + 9 = 31.5$
(b) $x = 20$ is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)
Answer: (a) 31.5 (b) x = 20 is outside the range of the data

5 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 12^2 + 12^2 = 288$
$AG^2 = AC^2 + CG^2 = 288 + 14^2 = 484$
AG = 22 cm
(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{14}{\sqrt{288}}$
angle GAC = 39.5° (1 d.p.)
Answer: (a) 22 cm (b) 39.5°

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6 Iteration

Use the iteration formula $x_{n+1} = \sqrt[3]{2x_n + 7}$ with $x_0 = 4$ (Calculator)
to work out x_1, x_2 and x_3.
Give each answer correct to 3 decimal places.

$x_1 = \sqrt[3]{2 \times 4 + 7} = 2.46621...$

$x_2 = \sqrt[3]{2 \times x_1 + 7} = 2.28512...$

$x_3 = \sqrt[3]{2 \times x_2 + 7} = 2.26177...$

Rounded: $x_1 = 2.466$, $x_2 = 2.285$, $x_3 = 2.262$

Answer: $x_1 = 2.466$, $x_2 = 2.285$, $x_3 = 2.262$

7 Inverse proportion

T is inversely proportional to the cube of n.
When n = 1, T = 64.
Find the positive value of n when T = 1.

$T = \frac{k}{n^3}$

$64 = \frac{k}{1}$, so $k = 64 \times 1 = 64$

$1 = \frac{64}{n^3}$, so $n^3 = \frac{64}{1} = 64$

$n = \sqrt[3]{64} = 4$

Answer: n = 4

8 Equation of a circle

A circle has centre C(2, −3) and passes through the point P(1, −4).
Find an equation of the circle.

$r^2 = CP^2 = (1 - 2)^2 + (-4 - (-3))^2$

$= 1 + 1 = 2$

$(x - 2)^2 + (y + 3)^2 = 2$

Answer: $(x - 2)^2 + (y + 3)^2 = 2$

9 Independent events and probability rules

A and B are two events.
P(A) = 0.5, P(B) = 0.6 and P(A or B) = 0.95
(a) Work out P(A and B).
(b) Are A and B independent? You must show your working.

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.95 = 0.5 + 0.6 - P(A \cap B)$, so $P(A \cap B) = 0.15$

(b) $P(A) \times P(B) = 0.5 \times 0.6 = 0.3$

This is not equal to 0.15, so A and B are not independent

Answer: (a) 0.15 (b) No

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10 Loci and regions

ABCD is a rectangle with AB = 4 cm and AD = 8 cm.
A point P inside the rectangle is such that AP ≤ 4 cm and P is nearer to A than to B.
Work out the area of the region in which P can lie. Give your answer in terms of π and surds.

Nearer to A than to B: P is on the A side of the perpendicular bisector of AB, a distance 2 cm from A
AP ≤ 4 cm: P is inside the quarter circle of radius 4 cm centred at A

The bisector meets the arc where $\cos\theta = \frac{2}{4}$, so $\theta = 60°$ at A. The region is a right-angled triangle plus a sector of angle 30°

Triangle: base 2 cm, height $4\sin 60° = 2\sqrt{3}$ cm, area $= \frac{1}{2} \times 2 \times 2\sqrt{3} = 2\sqrt{3}$

Sector of angle 30° : $\frac{30}{360} \times \pi \times 4^2 = \frac{4}{3}\pi$

Area $= 2\sqrt{3} + \frac{4}{3}\pi$ cm^2

Answer: $(2\sqrt{3} + \frac{4}{3}\pi)$ cm^2