Day 222 **S Tier 10-a-day: Model Solutions**

1 Estimating the median from grouped data

The table gives information about 96 times.
Work out an estimate for the median.
Assume the values in each class are evenly spread.

Cumulative frequencies: 20, 44, 52, 66, 96
The median is the value at position 96 ÷ 2 = 48
It is in the class 40 to 60, with 44 values before it, so we need 4 more of the 8 values in this class

Median $\approx 40 + \frac{4}{8} \times 20 = 50$

Answer: 50

2 Vectors in a triangle

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 2. N is on AB with AN : NB = 2 : 1.
(a) Find ON in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Show that MN is parallel to OB.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{2}{3}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{2}{3}(\mathbf{b} - \mathbf{a}) = \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$

(b) OM = $\frac{1}{3}\mathbf{a}$

MN = ON − OM = $\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b} - \frac{1}{3}\mathbf{a} = \frac{2}{3}\mathbf{b}$

MN = $\frac{2}{3}\mathbf{b}$, which is a multiple of OB, so MN is parallel to OB

Answer: (a) $\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$ (b) MN = $\frac{2}{3}\mathbf{b}$, parallel to OB

3 Ratio and mixtures

Container A holds juice and water in the ratio 5 : 2.
Container B holds juice and water in the ratio 2 : 2.
56 litres from container A is mixed with some of container B.
The ratio of juice to water in the mixture is 7 : 4.
Work out how many litres were taken from container B.

Let y litres be taken from B

juice: $40 + \frac{2y}{4}$

water: $16 + \frac{2y}{4}$

Ratio 7 : 4, so $4(40 + \frac{2y}{4}) = 7(16 + \frac{2y}{4})$

$(\frac{-3}{2})y = -48$, so y = 32

Answer: 32 litres

4 Comparing costs

FitLife has a joining fee of £4.50 plus £5 per month.
GymNow has a joining fee of £50 plus £1.50 per month.
(a) Work out the number of months for which the two costs are the same.
(b) Which is cheaper for 18 months, and by how much?

(a) Let n be the number of months. Working in pounds, the costs are equal when 4.50 + 5n = 50 + 1.50n
3.50n = 45.50, so n = 13
(b) FitLife: £4.50 + £5 × 18 = £94.50
GymNow: £50 + £1.50 × 18 = £77
GymNow is cheaper by £17.50
Answer: (a) 13 (b) GymNow, by £17.50

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5 Exact trigonometric values

Work out the exact value of tan 30° + tan 60°
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

tan 30° + tan 60° = $\frac{\sqrt{3}}{3} + \sqrt{3}$

= $\frac{4\sqrt{3}}{3}$

Answer: $\mathbf{\frac{4\sqrt{3}}{3}}$

6 Where a line meets the axes

The line L has equation $15x + 8y = -120$.
L crosses the x-axis at the point A and the y-axis at the point B.
(a) Find the coordinates of A and B.
(b) Work out the area of triangle OAB, where O is the origin.

(a) At A, $y = 0$: $15x = -120$, so $x = -8$. A is $(-8, 0)$
At B, $x = 0$: $8y = -120$, so $y = -15$. B is $(0, -15)$
(b) OA = 8 and OB = 15, and angle AOB = 90°

Area = $\frac{1}{2} \times 8 \times 15 = 60$

Answer: (a) A$(-8, 0)$, B$(0, -15)$ (b) 60

7 Recurring decimals

Write $0.\dot{8}$ as a fraction in its simplest form.

Let x = 0.8888...
10x = 8.8888...
Subtract: 9x = 8
x = $\frac{8}{9}$

Answer: $\mathbf{\frac{8}{9}}$

8 Exact lengths and areas

A rectangle has area (11) cm^2 and length $(4 + \sqrt{5})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{5}$

Width = area ÷ length = $\frac{11}{4 + \sqrt{5}}$

Multiply top and bottom by $(4 - \sqrt{5})$:
Numerator: $(11)(4 - \sqrt{5}) = 44 - 11\sqrt{5}$
Denominator: $16 - 5 = 11$
Width = $4 - \sqrt{5}$

Answer: $4 - \sqrt{5}$ cm

9 Tangent to a circle: proof

Show that the line $y = x + 8$ is a tangent to the circle $x^2 + y^2 = 32$.
Find the coordinates of the point where the line touches the circle.

Substitute $y = x + 8$ into $x^2 + y^2 = 32$: $x^2 + (x + 8)^2 = 32$
Expand: $2x^2 + 16x + 32 = 0$
$b^2 - 4ac = 16^2 - 4 \times 2 \times 32 = 256 - 256 = 0$
$b^2 - 4ac = 0$, so there is exactly one point of intersection: the line is a tangent
$x = \frac{-16}{2 \times 2} = -4$, and $y = (-4) + 8 = 4$. The point is $(-4, 4)$

Answer: $(-4, 4)$

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10 Fractional and negative indices

Work out $(\frac{16}{81})^{-\frac{1}{4}}$

A negative power means take the reciprocal: $(\frac{16}{81})^{-\frac{1}{4}} = (\frac{81}{16})^{\frac{1}{4}}$

The denominator 4 means the fourth root: the fourth root of $\frac{81}{16}$ is $\frac{3}{2}$

The numerator 1 means raise to the power 1: $(\frac{3}{2})^1 = \frac{3}{2}$

Answer: $\mathbf{\frac{3}{2}}$