Day 220 **S Tier 10-a-day: Model Solutions**

1 Prime factors and indices

Write $27^4 \times 6^3$ as a product of powers of prime numbers.

$27 = 3^3$, $6 = 2 \times 3$

$27^4 = 3^{12}$

$6^3 = 2^3 \times 3^3$

Add the powers of each prime: $2^3 \times 3^{15}$

Answer: $2^3 \times 3^{15}$

2 Mean of a changing set

The mean of 9 numbers is 7.
One number is removed and the mean of the remaining 8 numbers is 2.25.
Work out the number that was removed.

Total of the 9 numbers = $9 \times 7 = 63$
Total of the 8 remaining = $8 \times 2.25 = 18$
Number removed = $63 - 18 = 45$
Answer: 45

3 Comparing costs

Zip Taxis has a fixed charge of £5 plus £5.50 per mile.
Go Cabs has a fixed charge of £20 plus £4.50 per mile.
(a) Work out the number of miles for which the two costs are the same.
(b) Which is cheaper for 22 miles, and by how much?

(a) Let n be the number of miles. Working in pounds, the costs are equal when $5 + 5.50n = 20 + 4.50n$
$n = 15$
(b) Zip Taxis: £5 + £5.50 × 22 = £126
Go Cabs: £20 + £4.50 × 22 = £119
Go Cabs is cheaper by £7
Answer: (a) 15 (b) Go Cabs, by £7

4 Similar shapes

Two statues are mathematically similar.
The heights of the statues are 10 cm and 35 cm.
The smaller one has volume 200 cm^3.
Work out the volume of the larger one.

Scale factor of lengths = $35 \div 10 = \frac{7}{2}$

Scale factor of volumes = $(\frac{7}{2})^3 = \frac{343}{8}$

$200 \times \frac{343}{8} = 8575\ cm^3$

Answer: 8575 cm^3

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5 Fractions of fractions

Kofi cycles $\frac{7}{10}$ of a route before lunch.

After lunch he cycles $\frac{4}{9}$ of the remaining distance.

He still has 30 km to go.
How long is the route, in km?

After the first part, $\frac{3}{10}$ remains

After the second, $\frac{5}{9}$ of $\frac{3}{10}$ remains = $\frac{1}{6}$ of the total

$\frac{1}{6}$ of the total is 30

Total = $30 \div \frac{1}{6} = 180$

Answer: 180 km

6 Iteration

Use the iteration formula $x_{n+1} = 4 + \frac{3}{x_n}$ with $x_0 = 1$ (Calculator)

to work out x_1, x_2 and x_3.
Give each answer correct to 3 decimal places.

$x_1 = 4 + \frac{3}{1} = 7.00000...$

$x_2 = 4 + \frac{3}{x_1} = 4.42857...$

$x_3 = 4 + \frac{3}{x_2} = 4.67742...$

Rounded: $x_1 = 7.000$, $x_2 = 4.429$, $x_3 = 4.677$

Answer: $x_1 = 7.000$, $x_2 = 4.429$, $x_3 = 4.677$

7 Weighted percentages

The ratio of men to women in a school is 4 : 1.
25% of the men and 85% of the women own a car.
What percentage of all of these students own a car?

Imagine 100 students in total: 80 men and 20 women
25% of 80 = 20 and 85% of 20 = 17
Together: 37 out of 100
37 ÷ 100 × 100 = 37%
Answer: 37%

8 Angles of elevation

A point A is on level ground in line with the base of a mast. (Calculator)
The angle of elevation of the top of the mast from A is 54°.
B is a point on the same line, 75 m further from the mast than A.
The angle of elevation of the top of the mast from B is 38°.
Work out the height of the mast. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 54° = \frac{h}{x}$, so $x = h \div \tan 54°$

From B: $\tan 38° = \frac{h}{x + 75}$, so $x + 75 = h \div \tan 38°$

Subtract: $75 = h \div \tan 38° - h \div \tan 54° = h(1.2799 - 0.72654)$
$h = 75 \div 0.55340 = 135.5$ m (1 d.p.)
Answer: 135.5 m

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9 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 + x - 12}{x + 6} \div \frac{x - 3}{x^2 + 8x + 12}$

Factorise: $\frac{(x + 4)(x - 3)}{(x + 6)} \div \frac{(x - 3)}{(x + 6)(x + 2)}$

Turn the second fraction upside down and multiply: $\frac{(x + 4)(x - 3)}{(x + 6)} \times \frac{(x + 6)(x + 2)}{(x - 3)}$

Cancel $(x - 3)$ and $(x + 6)$
$= (x + 2)(x + 4)$
Answer: $(x + 2)(x + 4)$

10 Recasting solids

A solid metal sphere of radius 9 cm is melted down and recast into a solid cylinder of radius 6 cm.
Work out the height of the cylinder.

The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

Volume of sphere = $\frac{4}{3} \times \pi \times 9^3 = 972\pi$

Volume of cylinder = $\pi \times 6^2 \times h = 36\pi h$
$36h = 972$
$h = 27$ cm
Answer: 27 cm