Day 22 **S Tier 10-a-day: Model Solutions**

1 Weighted percentages

The ratio of juniors to seniors in a school is 1 : 4.
65% of the juniors and 70% of the seniors play a sport.
What percentage of all of these students play a sport?

Imagine 100 students in total: 20 juniors and 80 seniors
65% of 20 = 13 and 70% of 80 = 56
Together: 69 out of 100
69 ÷ 100 × 100 = 69%
Answer: 69%

2 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 4^2 + 8^2 = 80$
$AG^2 = AC^2 + CG^2 = 80 + 1^2 = 81$
AG = 9 cm

(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{1}{\sqrt{80}}$

angle GAC = 6.4° (1 d.p.)
Answer: (a) 9 cm (b) 6.4°

3 Inequalities

(a) Solve $-2 - 3x \geq 10$
(b) n is an integer. Write down all the values of n that satisfy
$9 \leq 2n + 5 \leq 19$

(a) Add 2: $-3x \geq 12$
Divide by -3 and reverse the inequality sign: $x \leq -4$
(b) Subtract 5: $4 \leq 2n \leq 14$
Divide by 2: $2 \leq n \leq 7$
Integers: 2, 3, 4, 5, 6, 7
Answer: (a) $x \leq -4$ (b) 2, 3, 4, 5, 6, 7

4 Arc length and sector area

A sector of a circle has radius 15 cm and angle 135°.
(a) Work out the length of the arc of the sector.
(b) Work out the area of the sector. Give each answer in terms of π.

(a) Fraction of the circle = $\frac{135}{360}$

Arc length = $\frac{135}{360} \times 2 \times \pi \times 15 = \frac{45}{4}\pi$ cm

(b) Area = $\frac{135}{360} \times \pi \times 15^2 = \frac{675}{8}\pi$ cm^2

Answer: (a) $\frac{45}{4}\pi$ cm (b) $\frac{675}{8}\pi$ cm^2

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5 Upper and lower bounds

a = 3.3, b = 2 and c = 4.5,
all correct to 1 decimal place.

Work out the upper bound of $\frac{a + b}{c}$.

Give your answer correct to 3 significant figures.

Each value is rounded, so it can be up to 0.05 above or below
$3.25 \leq a < 3.35$; $1.95 \leq b < 2.05$; $4.45 \leq c < 4.55$
Biggest answer: (biggest a + biggest b) ÷ smallest c = (3.35 + 2.05) ÷ 4.45
= 1.21348... = 1.21 to 3 significant figures
Answer: 1.21

6 Simultaneous equations

Solve the simultaneous equations
$6x + 3y = 3$
$-5x + 6y = 23$

To match the x terms: (1) × 5 gives $30x + 15y = 15$, and (2) × 6 gives $-30x + 36y = 138$
Add the equations to eliminate x: $51y = 153$
$y = 3$
Substitute y = 3 into (1): $6x + 9 = 3$
$6x = -6$, so $x = -1$
Answer: x = −1, y = 3

7 Prime factors and indices

Write $20^3 \times 30^4$ as a product of powers of prime numbers.

$20 = 2^2 \times 5$, $30 = 2 \times 3 \times 5$
$20^3 = 2^6 \times 5^3$
$30^4 = 2^4 \times 3^4 \times 5^4$
Add the powers of each prime: $2^{10} \times 3^4 \times 5^7$
Answer: $2^{10} \times 3^4 \times 5^7$

8 Reverse percentages

The price of a games console is increased by 40%.
Later the new price is reduced by 40% in a sale.
The sale price is £403.20.
Work out the price before the increase. (Calculator)

Multipliers: 1.4 and 0.6
Overall multiplier = 1.4 × 0.6 = 0.84
Original × 0.84 = £403.20
Original = £403.20 ÷ 0.84 = £480
Answer: £480

9 Terms of a quadratic sequence

The nth term of a sequence is $n^2 + 8n - 3$.
(a) Work out the 12th term.
(b) Is 382 a term of the sequence? You must show your working.

(a) $12^2 + 8 \times 12 - 3 = 237$
(b) Solve $n^2 + 8n - 3 = 382$, so $n^2 + 8n - 385 = 0$
$b^2 - 4ac = 8^2 - 4 \times 1 \times (-385) = 1604$, which is not a square number
So n is not a whole number, and 382 is not a term
Answer: (a) 237 (b) No

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10 Nested surds

Show that $\sqrt{88 + 30\sqrt{7}} = 5 + 3\sqrt{7}$

Square the right-hand side: $(5 + 3\sqrt{7})^2 = 25 + 30\sqrt{7} + 63$

$= 88 + 30\sqrt{7}$

So $\sqrt{88 + 30\sqrt{7}} = 5 + 3\sqrt{7}$

Answer: $\boldsymbol{\sqrt{88 + 30\sqrt{7}} = 5 + 3\sqrt{7}}$