Day 219 **S Tier 10-a-day: Model Solutions**

1 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve. It crosses the axes at the points shown.
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The roots are $x = -6$ and $x = 2$, so $y = a(x + 6)(x - 2)$
When $x = 0$, $y = 12$: $a \times 6 \times (-2) = 12$, so $a = -1$
$y = -(x + 6)(x - 2)$
$y = -x^2 - 4x + 12$
Answer: $y = -x^2 - 4x + 12$

2 Independent events and probability rules

A and B are two events.
P(A) = 0.6, P(B) = 0.7 and P(A or B) = 0.95
(a) Work out P(A and B).
(b) Are A and B independent? You must show your working.

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.95 = 0.6 + 0.7 - P(A \cap B)$, so $P(A \cap B) = 0.35$
(b) $P(A) \times P(B) = 0.6 \times 0.7 = 0.42$
This is not equal to 0.35, so A and B are not independent
Answer: (a) 0.35 (b) No

3 Laws of indices

Given that $\frac{25^5 \times 25^4}{5^2} = 5^n$,
find the value of n.

$25 = 5^2$
Numerator: $5^{10} \times 5^8 = 5^{18}$
Denominator: 5^2
Subtract the powers: $18 - 2 = 16$
Answer: n = 16

4 Probabilities with an unknown

A biased wheel stops on one of four outcomes with these probabilities.
A: 3x; B: x + 0.1; C: x + 0.1; D: 0.55
(a) Work out the value of x.
(b) The biased wheel is used 40 times. Work out an estimate for the number of times it stops on A.

(a) Probabilities add up to 1: $3x + x + 0.1 + x + 0.1 + 0.55 = 1$
$5x + 0.75 = 1$, so $5x = 0.25$
$x = 0.05$
(b) $P(A) = 3 \times 0.05 = 0.15$, so $40 \times 0.15 = 6$
Answer: (a) x = 0.05 (b) 6

5 Algebraic proof

Prove that the product of any two consecutive even numbers is a multiple of 8.

Let the even numbers be $2n$ and $2n + 2$, where n is an integer.
$2n(2n + 2) = 4n^2 + 4n = 4n(n + 1)$
n and n + 1 are consecutive integers, so one of them is even, and $n(n + 1) = 2 \times$ an integer
So $4n(n + 1) = 8 \times$ an integer, a multiple of 8
Answer: Proved

Day 219 **S Tier 10-a-day: Model Solutions**

6 Reverse percentages

The price of a laptop is increased by 25%.
The new price is then decreased by 12%.
The new price is then increased by 15%.
The final price is £1,416.80.
Work out the original price of the laptop.

Multipliers: 1.25, 0.88, 1.15
Overall multiplier = 1.25 × 0.88 × 1.15 = 1.265
Original = final ÷ multiplier = £1,416.80 ÷ 1.265
= £1,120
Answer: £1,120

7 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the opposite side BC and the known side is the hypotenuse AC, so use sin 60°

$\sin 60° = \frac{\sqrt{3}}{2}$

$x = 2 \times \sin 60° = 2 \times \frac{\sqrt{3}}{2}$

$x = \sqrt{3}$
Answer: $x = \sqrt{3}$

8 Equations in disguise

Solve $x^4 - 45x^2 + 324 = 0$

Let $u = x^2$: $u^2 - 45u + 324 = 0$
Factorise: $(u - 36)(u - 9) = 0$, so $u = 36$ or $u = 9$
$x^2 = 36$ or $x^2 = 9$
$x = \pm 6$ or $x = \pm 3$
Answer: $x = -6, -3, 3, 6$

9 Transformations of graphs

The graph of $y = f(x)$ has a maximum point at $(-3, -4)$.
Write down the coordinates of the turning point of the graph of
(a) $y = f(x + 4)$
(b) $y = f(x - 1) + 5$

(a) Start with the turning point $(-3, -4)$
Apply the transformation to the point: $(-7, -4)$
(b) $f(x - 1)$ changes the x-coordinate to -2, and $f(x) + 5$ changes the y-coordinate to 1
Turning point: $(-2, 1)$
Answer: (a) $(-7, -4)$ (b) $(-2, 1)$

10 Squares, cubes and prime factors

Find the smallest positive integer k such that $21 \times k$ is a square number.

$21 = 3 \times 7$
For a square number every power must be a multiple of 2
Multiply by 3×7 to get $3^2 \times 7^2$
$k = 21$
Answer: $k = 21$