Day 218 **S Tier 10-a-day: Model Solutions**

1 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve. It crosses the axes at the points shown.
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The roots are $x = -3$ and $x = 2$, so $y = a(x + 3)(x - 2)$
When $x = 0$, $y = -6$: $a \times 3 \times (-2) = -6$, so $a = 1$
$y = (x + 3)(x - 2)$
$y = x^2 + x - 6$
Answer: $y = x^2 + x - 6$

2 Solving trigonometric equations

Solve $\tan x = -\frac{\sqrt{3}}{3}$ for $0° \leq x \leq 360°$.
Do not use a calculator.

The related acute angle satisfies $\tan \theta = \frac{\sqrt{3}}{3}$, so $\theta = 30°$
tangent is negative in the 2nd and 4th quadrants, so $x = 180° - 30°$ and $x = 360° - 30°$
$x = 150°$ and $x = 330°$
Answer: $x = 150°$ or $x = 330°$

3 Percentage change and proportion

The volume of a cube is directly proportional to the cube of the length of its edge.
The length of its edge is increased by 100%.
Work out the percentage change in the volume of a cube.

The length of its edge is multiplied by 2
The multiplier for the volume of a cube is $2^3 = 8$
8 is an increase of 700%
Answer: 700% increase

4 Area of a triangle using sine

Triangle ABC has AB = 16 cm and BC = 10 cm.
The area of triangle ABC is 40 cm^2.
Angle ABC is acute.
Work out the size of angle ABC.
Do not use a calculator.

Area $= \frac{1}{2} \times AB \times BC \times \sin B$

$40 = \frac{1}{2} \times 16 \times 10 \times \sin B$

$\sin B = 40 \div 80 = \frac{1}{2}$

$\sin 30° = \frac{1}{2}$, and B is acute, so $B = 30°$

Answer: 30°

5 Standard form

Work out $(8.55 \times 10^{-6}) \div (1.5 \times 10^{-1})$
Give your answer in standard form.

$8.55 \div 1.5 = 5.7$
$10^{-6} \div 10^{-1} = 10^{-5}$
$= 5.7 \times 10^{-5}$
Answer: 5.7×10^{-5}

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6 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 + x - 20}{x^2 + 6x} \times \frac{x^2 - 3x}{x - 4}$

Factorise: $\frac{(x - 4)(x + 5)}{x(x + 6)} \times \frac{x(x - 3)}{(x - 4)}$

Cancel $(x - 4)$ and x

$= \frac{(x - 3)(x + 5)}{(x + 6)}$

Answer: $\mathbf{\frac{(x - 3)(x + 5)}{(x + 6)}}$

7 Recurring decimals

Convert the recurring decimal $0.42\dot{9}\dot{3}$ to a fraction.
Give your fraction in its simplest form.

Let $x = 0.4293...$
$100x = 42.9393...$
$10{,}000x = 4293.9393...$
Subtract: $9900x = 4251$
$x = \frac{4251}{9900} = \frac{1417}{3300}$

Answer: $\mathbf{\frac{1417}{3300}}$

8 Laws of indices

Write $\frac{2^{n+2} \times 8^n}{4^n}$ in the form $2^{an + c}$

where a and c are integers.

$2 = 2^1$, $8 = 2^3$, $4 = 2^2$
$2^{n+2} = 2^{n+2}$, $8^n = 2^{3(n)}$, $4^n = 2^{2n}$
Exponent: $(n + 2) + 3(n) - 2n = 2n + 2$
Answer: a = 2, c = 2

9 Probability from a frequency tree

In a population of 20,000 people, 8% have a certain disease.
A test gives a positive result for 80% of people who have the disease and for 1% of people who do not.
A person is chosen at random and tests positive.
Work out the probability that they have the disease.

8% of 20,000 = 1,600 have the disease and 18,400 do not
Flagged and genuine: 1,600 × 80% = 1,280
Flagged but not: 18,400 × 1% = 184
Total flagged or positive = 1,280 + 184 = 1,464
Probability = 1,280 ÷ 1,464 = $\frac{160}{183}$

Answer: $\mathbf{\frac{160}{183}}$

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10 Angles of elevation

A point A is on level ground in line with the base of a lighthouse. (Calculator)
The angle of elevation of the top of the lighthouse from A is 35°.
B is a point on the same line, 80 m further from the lighthouse than A.
The angle of elevation of the top of the lighthouse from B is 18°.
Work out the height of the lighthouse. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 35° = \frac{h}{x}$, so $x = h \div \tan 35°$

From B: $\tan 18° = \frac{h}{x + 80}$, so $x + 80 = h \div \tan 18°$

Subtract: $80 = h \div \tan 18° - h \div \tan 35° = h(3.0777 - 1.4281)$

$h = 80 \div 1.6495 = 48.5$ m (1 d.p.)

Answer: 48.5 m