Day 217 **S Tier 10-a-day: Model Solutions**

1 Equations with fractions

Solve $\frac{x+7}{3} + \frac{x+3}{8} = 5$

Multiply every term by 24: $8(x + 7) + 3(x + 3) = 120$
Expand: $11x + 65 = 120$
$11x = 55$
$x = 5$
Answer: x = 5

2 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the hypotenuse AC and the known side is the adjacent side AB, so use cos 60°

$\cos 60° = \frac{1}{2}$

$x = 7 \div \cos 60° = 7 \div \frac{1}{2}$

$x = 14$
Answer: x = 14

3 Comparing box plots

The box plots show the test marks of the students in two classes, A and B.
(a) Work out the interquartile range for each.
(b) Compare the distributions of A and B.

(a) IQR of A = 40 − 20 = 20
IQR of B = 90 − 45 = 45
(b) Class B has the higher median (50 compared with 30), so on average Class B scored higher
Class A has the smaller interquartile range (20 compared with 45), so its marks are more consistent
Answer: (a) A: 20, B: 45 (b) B has the higher median, A has the smaller IQR

4 Exact lengths and areas

A rectangle has area $(22 + 10\sqrt{5})$ cm^2 and length $(4 + 2\sqrt{5})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{5}$

Width = area ÷ length = $\frac{22 + 10\sqrt{5}}{4 + 2\sqrt{5}}$

Multiply top and bottom by $(4 - 2\sqrt{5})$:
Numerator: $(22 + 10\sqrt{5})(4 - 2\sqrt{5}) = -12 - 4\sqrt{5}$
Denominator: $16 - 20 = -4$
Width = $3 + \sqrt{5}$
Answer: 3 + $\sqrt{5}$ cm

5 Bounds in geometry

A runner covers 250 metres, correct to the nearest 50 metres, in a time of 27.2 seconds, correct to 1 decimal place. (Calculator)
Work out the upper bound of the runner's average speed in metres per second. Give your answer correct to 3 significant figures.

Distance is between 225 and 275 metres; time is between 27.15 and 27.25 seconds
Speed = distance ÷ time. For the upper bound use the greatest distance and the smallest time
275 ÷ 27.15 = 10.1 m/s (3 s.f.)
Answer: 10.1 m/s

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6 Cumulative frequency graphs

The cumulative frequency graph gives information about the marks of 100 students in a test.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 100 had a mark greater than 45.

(a) Median at cumulative frequency 50, so median = 30
Lower quartile at 25, so LQ = 20. Upper quartile at 75, so UQ = 40
Interquartile range = 40 − 20 = 20
(b) At 45, cumulative frequency = 81
Number greater = 100 − 81 = 19
Answer: (a) median 30, IQR 20 (b) 19

7 Line and curve intersections

Solve the simultaneous equations
$y = x^2 + x - 1$
$y = -2x + 3$

Substitute: $x^2 + x - 1 = -2x + 3$
Rearrange: $x^2 + 3x - 4 = 0$
Factorise: $(x + 4)(x - 1) = 0$, so $x = -4$ or $x = 1$
When $x = -4$: $y = -2 \times (-4) + 3 = 11$
When $x = 1$: $y = -2 \times 1 + 3 = 1$
Answer: $x = -4$, $y = 11$ and $x = 1$, $y = 1$

8 Functions that are their own inverse

$f(x) = \frac{-5x - 1}{x + 5}$ where $x \neq -5$
(a) Show that $ff(x) = x$.
(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by (x + 5)
$ff(x) = \frac{(-5)(-5x - 1) + (-1)(x + 5)}{(-5x - 1) - (-5)(x + 5)}$
Top = 25x + 5 − x − 5 = 24x
Bottom = −5x − 1 + 5x + 25 = 24
$ff(x) = \frac{24x}{24} = x$

(b) ff(x) = x, so f is its own inverse: $f^{-1}(x) = \frac{-5x - 1}{x + 5}$

Answer: (b) $f^{-1}(x) = \frac{-5x - 1}{x + 5}$

9 Loci and regions

ABCD is a rectangle with AB = 10 cm and AD = 14 cm.
A point P inside the rectangle is such that AP ≤ 8 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 8 cm: P is inside the quarter circle of radius 8 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves
Area $= \frac{1}{2} \times \frac{1}{4} \times \pi \times 8^2 = 8\pi$ cm^2

Answer: 8π cm^2

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10 Surds and fractions

Simplify $\frac{3}{4+\sqrt{10}} - \frac{1}{4-\sqrt{10}}$

Give your answer with a rational denominator.

Common denominator: $(4+\sqrt{10})(4-\sqrt{10}) = 16 - 10 = 6$

Numerator: $3(4-\sqrt{10}) - (4+\sqrt{10})$

$= 8 - 4\sqrt{10}$

$\frac{8-4\sqrt{10}}{6} = \frac{4-2\sqrt{10}}{3}$

Answer: $\mathbf{\frac{4-2\sqrt{10}}{3}}$