Day 216 **S Tier 10-a-day: Model Solutions**

1 Error intervals

The capacity of a tank is 0.24 litres, truncated to 2 decimal places.
Write down the error interval for the capacity of a tank, using c.

Truncating cuts off the later digits without rounding.
The smallest possible value is 0.24 and the value must be less than 0.25
$0.24 \leq c < 0.25$
Answer: $0.24 \leq c < 0.25$

2 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle ACB = 50°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle APB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: 2 × 50 = 100°
(b) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angle APB = 360 − 90 − 90 − 100 = 80°
Answer: (a) 100° (b) 80°

3 Factorising fully

Factorise fully $36x^2 - 4y^2$

$36x^2 - 4y^2$ is a difference of two squares: $(6x)^2 - (2y)^2$
$= (6x - 2y)(6x + 2y)$
Answer: $(6x - 2y)(6x + 2y)$

4 Estimating the mean from a histogram table

The table gives information about the times of some people.
Work out an estimate for the mean time.
Give your answer correct to 1 decimal place. (Calculator)

Frequency = frequency density × class width: 12, 12, 34, 16
Mid-points: 5, 15, 30, 50
$\Sigma fm = 12\times5 + 12\times15 + 34\times30 + 16\times50 = 2060$, $\Sigma f = 74$
Mean = 2060 ÷ 74 = 27.8
Answer: 27.8

5 Solving trigonometric equations

Solve 7 sin x + 2 = 0 for $0° \leq x \leq 360°$. (Calculator)
Give your answers correct to 1 decimal place.

$\sin x = -\frac{2}{7}$

Related acute angle: $\sin^{-1}(\frac{2}{7}) = 16.6°$

sine is negative in the 3rd and 4th quadrants, so x = 180° + 16.6° and x = 360° − 16.6°
x = 196.6° and x = 343.4°
Answer: x = 196.6° or x = 343.4°

6 Simple and compound interest

Dina invests £1,000 for 5 years at 4.5% per year simple interest.
Beth invests £1,000 for 5 years at 4.5% per year compound interest.
Work out how much more interest Beth earns than Dina.
Give your answer to the nearest penny. (Calculator)

Simple interest: £1,000 × 4.5% × 5 = £225
Compound: £1,000 × 1.045^5 = £1,246.18
Compound interest = £1,246.18 − £1,000 = £246.18
Difference = £246.18 − £225 = £21.18
Answer: £21.18

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7 Percentage change and proportion

The time for a pendulum to swing is directly proportional to the square root of the length of the pendulum.
The length of the pendulum is increased by 21%.
Work out the percentage change in the time for a pendulum to swing.

The length of the pendulum is multiplied by 1.21

The multiplier for the time for a pendulum to swing is $\sqrt{1.21} = 1.1$

1.1 is an increase of 10%

Answer: 10% increase

8 Iteration: rearranging equations

(a) Show that the equation $x^3 + x - 24 = 0$ has a solution between $x = 2$ and $x = 3$.

(b) Show that the equation can be rearranged to give $x = \sqrt[3]{24 - x}$

(a) Let $f(x) = x^3 + x - 24$. $f(2) = 2^3 + 2 - 24 = -14$ and $f(3) = 3^3 + 3 - 24 = 6$

f(2) is negative and f(3) is positive. There is a change of sign, so there is a solution between 2 and 3

(b) Rearrange $x^3 + x - 24 = 0$: $x^3 = 24 - x$

Take the cube root of both sides: $x = \sqrt[3]{24 - x}$

Answer: Shown

9 Counting factors

How many factors does 140 have?
Include 1 and 140.

$140 = 2^2 \times 5 \times 7$

Add 1 to each power and multiply: $3 \times 2 \times 2 = 12$

Answer: 12

10 Tangency conditions

The line $y = kx - 5$ is a tangent to the curve $y = x^2 + 5$.
Find the possible values of k.
Give any surds in their simplest form.

Equate: $x^2 + 5 = kx - 5$

Rearrange: $x^2 - kx + 10 = 0$

One point of contact, so $b^2 - 4ac = 0$: $k^2 - 4 \times 10 = 0$

$k^2 = 40$

$k = \pm\sqrt{40} = \pm 2\sqrt{10}$

Answer: $k = \pm 2\sqrt{10}$