Day 215 **S Tier 10-a-day: Model Solutions**

1 Substitution

Work out the value of $a^2 - 3bc$
when $a = -2$, $b = -3$ and $c = 2$.

Substitute the values: $(-2)^2 - 3 \times (-3) \times 2$
$= 4 - (-18)$
$= 22$
Answer: 22

2 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the radius of the circle.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) AM = 48 ÷ 2 = 24 cm
$OA^2 = OM^2 + AM^2 = 10^2 + 24^2 = 676$
OA = 26 cm

(b) sin(angle AOM) = $\frac{24}{26}$
Angle AOM = 67.38°, so angle AOB = 2 × 67.38 = 134.8°
Answer: (a) 26 cm (b) 134.8°

3 Direct proportion formulae

L is directly proportional to the square root of t.
When t = 16, L = 36.
(a) Find a formula for L in terms of t.
(b) Work out the positive value of t when L = 18.

(a) $L = k\sqrt{t}$
36 = k × 4, so k = 36 ÷ 4 = 9
$L = 9\sqrt{t}$
(b) $18 = 9\sqrt{t}$, so $\sqrt{t} = 18 \div 9 = 2$, giving t = 4
Answer: (a) $L = 9\sqrt{t}$ (b) t = 4

4 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle ACB = 42°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle OAB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: 2 × 42 = 84°
(b) OA = OB (radii), so triangle OAB is isosceles
Angle OAB = (180 − 84) ÷ 2 = 48°
Answer: (a) 84° (b) 48°

5 Standard form

Work out $(4.5 \times 10^4) \times (1.3 \times 10^8)$
Give your answer in standard form.

4.5 × 1.3 = 5.85
$10^4 \times 10^8 = 10^{12}$
$= 5.85 \times 10^{12}$
Answer: 5.85×10^{12}

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6 Histograms

The histogram shows the masses of parcels delivered by a courier.
(a) Work out the total number of parcels.
(b) Estimate the number of parcels with a mass between 15 kg and 65 kg.

Frequency = frequency density × class width
10 to 20: 0.4 × 10 = 4
20 to 40: 1.2 × 20 = 24
40 to 60: 1.6 × 20 = 32
60 to 70: 0.4 × 10 = 4
(a) Total = 64
(b) 15 to 20: 0.4 × 5 = 2
Whole bars between: 24 + 32 = 56
60 to 65: 0.4 × 5 = 2, so estimate = 60
Answer: (a) 64 (b) 60

7 Simplifying algebraic fractions

Simplify fully $\frac{x^2 - 8x + 7}{x^2 - 12x + 35}$

Factorise the top: $(x - 7)(x - 1)$
Factorise the bottom: $(x - 7)(x - 5)$
Cancel the common factor $(x - 7)$: $\frac{x - 1}{x - 5}$

Answer: $\mathbf{\frac{x - 1}{x - 5}}$

8 Perpendicular bisector

A is the point (8, 7) and B is the point (16, 3).
Find an equation of the perpendicular bisector of AB.
Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = $(\frac{8 + 16}{2}, \frac{7 + 3}{2}) = (12, 5)$

Gradient of AB = $\frac{3 - 7}{16 - 8} = -\frac{1}{2}$

The perpendicular gradient is $-1 \div (-\frac{1}{2}) = 2$

Using the midpoint: $y - 5 = 2(x - 12)$
Rearrange: $-2x + y = -19$
Multiply by -1: $2x - y = 19$
Answer: $2x - y = 19$

9 Fractional indices

Given that x > 0, solve $x^{-\frac{5}{2}} = \frac{1}{32768}$

Raise both sides to the power $-\frac{2}{5}$

$x = (\frac{1}{32768})^{-\frac{2}{5}}$
$x = 64$
Answer: x = 64

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10 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 2 cm, top radius 6 cm and base radius 12 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+2} = \frac{6}{12}$

$12h = 6(h + 2)$, so $6h = 12$ and $h = 2$

Large cone: $\frac{1}{3} \times \pi \times 12^2 \times 4 = 192\pi$

Small cone: $\frac{1}{3} \times \pi \times 6^2 \times 2 = 24\pi$

Frustum = $192\pi - 24\pi = 168\pi$ cm^3

Answer: 168π cm^3