Day 213 **S Tier 10-a-day: Model Solutions**

1 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 31°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 31°
Angle COB is an exterior angle of triangle OAC: 31 + 31 = 62°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 62°

2 Simplifying surds

Write $\sqrt{20} + \sqrt{125} + \sqrt{45}$ in the form $a\sqrt{5}$
where a is an integer.

$\sqrt{20} = 2\sqrt{5}$; $\sqrt{125} = 5\sqrt{5}$; $\sqrt{45} = 3\sqrt{5}$
$(2 + 5 + 3)\sqrt{5} = 10\sqrt{5}$
Answer: $10\sqrt{5}$

3 Angles with algebra

ABC is an isosceles triangle with AB = AC.
Angle BAC = (4x + 20)°
Angle ABC = (3x + 20)°
(a) Work out the value of x.
(b) Work out the size of angle BAC.

(a) Angles ABC and ACB are equal, and the angles in a triangle add up to 180°
(4x + 20) + 2(3x + 20) = 180
10x + 60 = 180, so 10x = 120
x = 12
(b) Angle BAC = 4 × 12 + 20 = 68°
Answer: (a) x = 12 (b) 68°

4 Exchange rates

Oscar changes £600 into euros at £1 = €1.35.
Oscar spends €40 and then changes the rest back into pounds at £1 = €1.6.
How many pounds does Oscar get back?

£600 × 1.35 = €810
810 − 40 = €770
770 ÷ 1.6 = £481.25
Answer: £481.25

5 Perimeter and area of a running track

The diagram shows a running track made from a rectangle and two semicircles.
(a) Work out the total length of the boundary of the track.
(b) Work out the area enclosed by the track. Give your answers in terms of π.
The diagram is not drawn accurately.

(a) The two semicircles make one full circle of radius 10: circumference = 2 × π × 10 = 20π
The two straight sides: 2 × 40 = 80
Boundary = (80 + 20π) m
(b) The rectangle is 40 m by 2 × 10 = 20 m, so its area is 40 × 20 = 800. Circle: $\pi \times 10^2 = 100\pi$
Area = (800 + 100π) m^2
Answer: (a) (80 + 20π) m (b) (800 + 100π) m^2

Day 213 **S Tier 10-a-day: Model Solutions**

6 Percentage change

A price is decreased by 50%.
By what percentage must the new value be increased to get back to the original value?

Let the original be 100. After a 50% decrease it is 50.
To return to 100, increase by 50

Percentage increase = $\frac{50}{50} \times 100 = 100\%$

Answer: 100%

7 Equations in disguise

Solve $x - 5\sqrt{x} + 6 = 0$

Let $u = \sqrt{x}$: $u^2 - 5u + 6 = 0$
Factorise: $(u - 3)(u - 2) = 0$, so $u = 3$ or $u = 2$
$\sqrt{x} = 3$ or $\sqrt{x} = 2$
$x = 9$ or $x = 4$
Answer: x = 4 or x = 9

8 Bounds of area and perimeter

A rectangle has length 30 cm and width 14 cm.
Both measurements are correct to the nearest centimetre.
Evan says that the perimeter of the rectangle could be exactly 91 cm.
Is Evan correct? You must show your working.

$29.5 \leq$ length < 30.5 and $13.5 \leq$ width < 14.5
Lower bound of perimeter $= 2 \times 29.5 + 2 \times 13.5 = 86$
Upper bound of perimeter $= 2 \times 30.5 + 2 \times 14.5 = 90$
So $86 \leq$ perimeter < 90
91 is outside this interval, so Evan is not correct
Answer: No, 91 is outside the interval

9 Estimated mean with an unknown frequency

The table gives information about the masses of some eggs.
An estimate for the mean is 18.8.
Work out the value of x.

Mid-points: 5, 15, 25, 35
Estimated mean $= \Sigma fm \div \Sigma f$, so $(745 + 15x) \div (37 + x) = 18.8$
$745 + 15x = 18.8(37 + x) = 695.6 + 18.8x$
$(15 - 18.8)x = 695.6 - 745$
$-3.8x = -49.4$, so $x = 13$
Answer: x = 13

10 Rearranging when the subject appears twice

Make x the subject of $y(x + 5) = x - y$

Expand the bracket: $yx + 5y = x - y$
Collect the x terms: $yx - x = -5y - y$
Factorise: $x(y - 1) = -6y$

$x = \frac{6y}{1 - y}$

Answer: x = $\frac{\mathbf{6y}}{\mathbf{1 - y}}$