Day 212 **S Tier 10-a-day: Model Solutions**

1 Weighted percentages

The ratio of juniors to seniors in a school is 2 : 3.
60% of the juniors and 55% of the seniors play a sport.
What percentage of all of these students play a sport?

Imagine 100 students in total: 40 juniors and 60 seniors
60% of 40 = 24 and 55% of 60 = 33
Together: 57 out of 100
57 ÷ 100 × 100 = 57%
Answer: 57%

2 Time zones

A flight leaves London at 09:15 (London time).
It takes 8 hours 15 minutes.
When it is 12:00 in London it is 07:00 in New York.
At what time, in New York time, does the flight land? Say if it is on a different day.

Arrival in London time = 09:15 + 8 h 15 min = 17:30
New York is 5 hours behind London, so subtract 5 hours
12:30
Answer: 12:30 the same day

3 Area of a segment

The circle has centre O and radius 8 cm. A and B are points on the circle and angle AOB = 130°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{130}{360} \times \pi \times 8^2 = 72.606\ \text{cm}^2$

Area of triangle OAB = $\frac{1}{2} \times 8 \times 8 \times \sin 130° = 24.513\ \text{cm}^2$

Segment = sector − triangle = 72.606 − 24.513 = 48.092
= 48.1 cm^2 (3 s.f.)
Answer: 48.1 cm^2

4 Fractions

Work out $1\frac{1}{2} \times 1\frac{2}{3} - 1\frac{4}{5}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions
$\frac{3}{2} \times \frac{5}{3} = \frac{5}{2}$
$\frac{5}{2} - \frac{9}{5} = \frac{7}{10}$
Answer: $\frac{7}{10}$

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5 Vectors in a triangle

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 4. N is on AB with AN : NB = 4 : 1.
(a) Find ON in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Show that MN is parallel to OB.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{4}{5}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{4}{5}(\mathbf{b} - \mathbf{a}) = \frac{1}{5}\mathbf{a} + \frac{4}{5}\mathbf{b}$

(b) OM = $\frac{1}{5}\mathbf{a}$

MN = ON − OM = $\frac{1}{5}\mathbf{a} + \frac{4}{5}\mathbf{b} - \frac{1}{5}\mathbf{a} = \frac{4}{5}\mathbf{b}$

MN = $\frac{4}{5}\mathbf{b}$, which is a multiple of OB, so MN is parallel to OB

Answer: (a) $\frac{1}{5}\mathbf{a} + \frac{4}{5}\mathbf{b}$ (b) MN = $\frac{4}{5}\mathbf{b}$, parallel to OB

6 Comparing box plots

The box plots show the test marks of the students in two classes, A and B.
(a) Work out the interquartile range for each.
(b) Compare the distributions of A and B.

(a) IQR of A = 50 − 35 = 15
IQR of B = 85 − 55 = 30
(b) Class B has the higher median (75 compared with 45), so on average Class B scored higher
Class A has the smaller interquartile range (15 compared with 30), so its marks are more consistent
Answer: (a) A: 15, B: 30 (b) B has the higher median, A has the smaller IQR

7 Exact trigonometric ratios from a given ratio

x is an acute angle and tan x = $\frac{7}{12}$.
(a) Find the exact value of sin x.
(b) Find the exact value of cos x. Give each answer as a fraction or a surd with a rational denominator.

Draw a right-angled triangle with opposite side 7 and adjacent side 12

Hypotenuse = $\sqrt{7^2 + 12^2} = \sqrt{193}$

(a) sin x = $\frac{7}{\sqrt{193}} = \frac{7\sqrt{193}}{193}$

(b) cos x = $\frac{12}{\sqrt{193}} = \frac{12\sqrt{193}}{193}$

Answer: (a) $\frac{7\sqrt{193}}{193}$ (b) $\frac{12\sqrt{193}}{193}$

8 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 + 2x - 3}{x^2 - 4x} \times \frac{x^2 + 2x}{x - 1}$

Factorise: $\frac{(x - 1)(x + 3)}{x(x - 4)} \times \frac{x(x + 2)}{(x - 1)}$

Cancel (x − 1) and x

= $\frac{(x + 2)(x + 3)}{(x - 4)}$

Answer: $\frac{(x + 2)(x + 3)}{(x - 4)}$

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9 Exact lengths and areas

A rectangle has area $(10 + 6\sqrt{3})$ cm^2 and length $(2 + \sqrt{3})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{3}$

Width = area ÷ length = $\dfrac{10 + 6\sqrt{3}}{2 + \sqrt{3}}$

Multiply top and bottom by $(2 - \sqrt{3})$:
Numerator: $(10 + 6\sqrt{3})(2 - \sqrt{3}) = 2 + 2\sqrt{3}$
Denominator: $4 - 3 = 1$
Width = $2 + 2\sqrt{3}$
Answer: $2 + 2\sqrt{3}$ cm

10 Circle and line intersections

Solve the simultaneous equations
$x^2 + y^2 = 50$
$y = 3x + 10$

Substitute into the first equation: $x^2 + (3x + 10)^2 = 50$
Expand: $10x^2 + 60x + 50 = 0$
Divide by 10: $x^2 + 6x + 5 = 0$
Factorise: $(x + 5)(x + 1) = 0$, so $x = -5$ or $x = -1$
$y = 3x + 10$ gives $y = -5$ and $y = 7$
Answer: $x = -5$, $y = -5$ and $x = -1$, $y = 7$