Day 211 **S Tier 10-a-day: Model Solutions**

1 Column vectors

$\mathbf{a} = \begin{pmatrix}2\\2\end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix}5\\3\end{pmatrix}$

Work out the exact length of the vector $\mathbf{a} - 2\mathbf{b}$. Give your answer as a simplified surd.

$\mathbf{a} - 2\mathbf{b} = \begin{pmatrix}2\\2\end{pmatrix} - 2\begin{pmatrix}5\\3\end{pmatrix} = \begin{pmatrix}-8\\-4\end{pmatrix}$

Length $= \sqrt{(-8)^2 + (-4)^2} = \sqrt{80}$

$= 4\sqrt{5}$

Answer: $4\sqrt{5}$

2 Stratified sampling

The table shows the number of students in each of four groups.

(a) A stratified sample of 38 is taken. Work out how many come from Year 10.

(b) A different stratified sample has 30 from Year 8. Work out the size of this sample.

Total = 200 + 200 + 180 + 180 = 760

(a) Year 10: $\frac{180}{760} \times 38 = 9$

(b) Fraction sampled = 30 ÷ 200 = $\frac{3}{20}$, so sample size = $\frac{3}{20} \times 760 = 114$

Answer: (a) 9 (b) 114

3 Completing the square

(a) Write $x^2 - 6x + 18$ in the form $(x + p)^2 + q$

(b) Write down the coordinates of the turning point of the curve $y = x^2 - 6x + 18$

(a) Half of −6 is −3, so $x^2 - 6x = (x - 3)^2 - 9$

$x^2 - 6x + 18 = (x - 3)^2 - 9 + 18$

$= (x - 3)^2 + 9$

(b) The minimum is when $x - 3 = 0$, so $x = 3$ and $y = 9$

Answer: (a) $(x - 3)^2 + 9$ (b) (3, 9)

4 Income tax

In one country, income tax is worked out like this:

The first £12,500 of income is not taxed.

The next £37,500 is taxed at 20%.

All income above that is taxed at 40%.

Kofi earns £86,000 in a year.

Work out how much income tax Kofi pays.

Taxed at 20%: £37,500

20% of £37,500 = £7,500

Taxed at 40%: £86,000 − £12,500 − £37,500 = £36,000

40% of £36,000 = £14,400

Total tax = £7,500 + £14,400 = £21,900

Answer: £21,900

5 Estimated mean with an unknown frequency

The table gives information about the times taken by some people to travel to work.

An estimate for the mean is 52.

Work out the value of x.

Mid-points: 10, 30, 50, 70, 90

Estimated mean = $\Sigma fm \div \Sigma f$, so $(2730 + 70x) \div (57 + x) = 52$

$2730 + 70x = 52(57 + x) = 2964 + 52x$

$(70 - 52)x = 2964 - 2730$

$18x = 234$, so $x = 13$

Answer: x = 13

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6 Bounds of area and perimeter

A rectangle has length 9 cm and width 13 cm.
Both measurements are correct to the nearest centimetre.
Uma says that the perimeter of the rectangle could be exactly 48 cm.
Is Uma correct? You must show your working.

$8.5 \leq$ length < 9.5 and $12.5 \leq$ width < 13.5
Lower bound of perimeter $= 2 \times 8.5 + 2 \times 12.5 = 42$
Upper bound of perimeter $= 2 \times 9.5 + 2 \times 13.5 = 46$
So $42 \leq$ perimeter < 46
48 is outside this interval, so Uma is not correct
Answer: No, 48 is outside the interval

7 Ratio and algebra

In a bag, the ratio of the number of red counters to the number of blue counters is 2 : 7.
Then 3 red counters are removed and 2 more blue counters are added.
The ratio is now 1 : 6.
How many blue counters were there at the start?

Let the numbers at the start be 2k and 7k
$(2k - 3) : (7k + 2) = 1 : 6$, so $6(2k - 3) = 1(7k + 2)$
$12k - 18 = 7k + 2$
$5k = 20$, so $k = 4$
Number of blue counters at the start: $7 \times 4 = 28$
Answer: 28

8 Equation of a circle

A circle has centre C(−5, 3) and passes through the point P(−1, 5).
Find an equation of the circle.

$r^2 = CP^2 = (-1 - (-5))^2 + (5 - 3)^2$
$= 16 + 4 = 20$
$(x + 5)^2 + (y - 3)^2 = 20$
Answer: $(x + 5)^2 + (y - 3)^2 = 20$

9 Loci and regions

ABCD is a rectangle with AB = 15 cm and AD = 16 cm.
A point P inside the rectangle is such that AP ≤ 12 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 12 cm: P is inside the quarter circle of radius 12 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves

Area $= \frac{1}{2} \times \frac{1}{4} \times \pi \times 12^2 = 18\pi$ cm^2

Answer: 18π cm^2

10 Circle and line intersections

Solve the simultaneous equations
$x^2 + y^2 = 65$
$y = 3x + 5$

Substitute into the first equation: $x^2 + (3x + 5)^2 = 65$
Expand: $10x^2 + 30x - 40 = 0$
Divide by 10: $x^2 + 3x - 4 = 0$
Factorise: $(x + 4)(x - 1) = 0$, so $x = -4$ or $x = 1$
$y = 3x + 5$ gives $y = -7$ and $y = 8$
Answer: $x = -4$, $y = -7$ and $x = 1$, $y = 8$