Day 210 **S Tier 10-a-day: Model Solutions**

1 Probabilities with an unknown

A biased spinner lands on one of four outcomes with these probabilities.
red: 2x; blue: 0.1; green: 0.2; yellow: 0.5
(a) Work out the value of x.
(b) The biased spinner is used 100 times. Work out an estimate for the number of times it lands on red.

(a) Probabilities add up to 1: $2x + 0.1 + 0.2 + 0.5 = 1$
$2x + 0.8 = 1$, so $2x = 0.2$
$x = 0.1$
(b) P(red) = $2 \times 0.1 = 0.2$, so $100 \times 0.2 = 20$
Answer: (a) x = 0.1 (b) 20

2 Quadratic graphs

The curve $y = -x^2 - 4x$ crosses the x-axis at the points A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the maximum point of the curve.

(a) Put $y = 0$. $-x^2 - 4x = 0$, so $-(x^2 - 4x) = 0$, giving $-(x + 4)(x) = 0$
$x + 4 = 0$ or $x = 0$, so $x = -4$ or $x = 0$
(b) The turning point is halfway between A and B: $x = (-4 + 0) \div 2 = -2$
When $x = -2$: $y = -1 \times (-2)^2 - 4 \times (-2) + 0 = 4$
Answer: (a) A(−4, 0), B(0, 0) (b) (−2, 4)

3 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $-\mathbf{a} - \mathbf{b}$, OQ = $-3\mathbf{a} - 7\mathbf{b}$ and OR = $-4\mathbf{a} - 10\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $-2\mathbf{a} - 6\mathbf{b}$
QR = OR − OQ = $-\mathbf{a} - 3\mathbf{b}$
QR = $\frac{1}{2}$ PQ, so PQ and QR are parallel, and they share the point Q
So P, Q and R lie on a straight line
(b) PQ : QR = 2 : 1
Answer: (b) 2 : 1

4 Quadratic inequalities

Solve $3x^2 - 6x - 45 < 0$

Divide by 3: $x^2 - 2x - 15 < 0$
Solve $x^2 - 2x - 15 = 0$: $(x + 3)(x - 5) = 0$, so $x = -3$ or $x = 5$
The graph is a U shape crossing the x-axis at −3 and 5; it is below the axis between the roots
$-3 < x < 5$
Answer: $-3 < x < 5$

5 Enlargement

Triangle P has vertices A(−3, −2), B(−5, −5) and C(0, −5).
Triangle Q has vertices A'(−3, 7), B'(1, 13) and C'(−9, 13).
Q is an enlargement of P.
Describe fully the single transformation that maps P onto Q.

x-step from A to B = −2, x-step from A' to B' = 4
Scale factor = $4 \div -2 = -2$
The centre C satisfies: $C + -2(A - C) = A'$, so $(1 - -2)C = A' - -2A$
x: $3 \times x = (-3) - (-2) \times (-3) = -9$, so $x = -3$
y: $3 \times y = 7 - (-2) \times (-2) = 3$, so $y = 1$
Answer: Enlargement, scale factor −2, centre (−3, 1)

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6 Speed and a moving train

A train travels at a constant speed of 108 km/h.
It takes 60 seconds to go completely through a tunnel which is 1,470 m long.
Work out the length of the train, in metres.

Speed = 108 km/h = 108,000 ÷ 3600 = 30 m/s
Distance travelled in 60 s = 30 × 60 = 1,800 m
This is the length of the tunnel plus the length of the train
Train = 1,800 − 1,470 = 330 m
Answer: 330 m

7 Reverse percentages

The value of a house is increased by 15% in one year.
The next year its value is increased by 12%.
The next year its value is increased by 25%.
It is now worth £595,700.
Work out the value of the house before the first change.

Multipliers: 1.15, 1.12, 1.25
Overall multiplier = 1.15 × 1.12 × 1.25 = 1.61
Original = final ÷ multiplier = £595,700 ÷ 1.61
= £370,000
Answer: £370,000

8 Shaded areas: exact answers

ABCD is a square of side 4 cm.
BD is an arc of a circle with centre A and an arc of a circle with centre C.
Work out the area of the shaded region. Give your answer in terms of π.

Area of quarter circle (centre A, radius 4) = $\frac{1}{4} \times \pi \times 4^2 = 4\pi$

The two quarter circles overlap in the shaded region and together cover the whole square
Shaded area = 2 × 4π − 16 = 8π − 16
Answer: $(8\pi - 16)$ cm^2

9 Iteration: rearranging equations

(a) Show that the equation $x^3 - 3x - 21 = 0$ has a solution between x = 3 and x = 4.
(b) Show that the equation can be rearranged to give $x = \sqrt[3]{3x + 21}$

(a) Let $f(x) = x^3 - 3x - 21$. $f(3) = 3^3 - 3 \times 3 - 21 = -3$ and $f(4) = 4^3 - 3 \times 4 - 21 = 31$
f(3) is negative and f(4) is positive. There is a change of sign, so there is a solution between 3 and 4
(b) Rearrange $x^3 - 3x - 21 = 0$: $x^3 = 3x + 21$
Take the cube root of both sides: $x = \sqrt[3]{3x + 21}$
Answer: Shown

10 Standard form

Work out $(7 \times 10^8) - (5.3 \times 10^5)$
Give your answer in standard form.

Write the first number as a multiple of 10^5: $7 \times 10^8 = 7000 \times 10^5$
$(7000 - 5.3) \times 10^5 = 6994.7 \times 10^5$
$= 6.9947 \times 10^8$
Answer: 6.9947×10^8