**Day 21** **S Tier 10-a-day: Model Solutions**

## 1 Simplifying algebraic fractions

Simplify fully $\frac{x^2 - 25}{x^2 + x - 20}$

Factorise the top: $(x - 5)(x + 5)$
Factorise the bottom: $(x + 5)(x - 4)$
Cancel the common factor $(x + 5)$: $\frac{x - 5}{x - 4}$

**Answer:** $\mathbf{\frac{x - 5}{x - 4}}$

## 2 Cyclic quadrilaterals

A, B, C and D are points on a circle. Angle DAB = (2x)°, angle BCD = (5x + 12)° and angle ABC = (3x + 25)°.
(a) Work out the value of x.
(b) Work out the size of angle ADC.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
(2x) + (5x + 12) = 180
7x + 12 = 180, so 7x = 168
x = 24
(b) Angle ABC = 3 × 24 + 25 = 97°
Angle ADC = 180 − 97 = 83°
**Answer: (a) x = 24 (b) 83°**

## 3 Weighted percentages

The ratio of Year 10 students to Year 11 students in a school is 5 : 1.
65% of the Year 10 students and 50% of the Year 11 students have a part-time job.
What percentage of all of these students have a part-time job?

Imagine 120 students in total: 100 Year 10 students and 20 Year 11 students
65% of 100 = 65 and 50% of 20 = 10
Together: 75 out of 120
75 ÷ 120 × 100 = 62.5%
**Answer: 62.5%**

## 4 Recurring decimals

Write $0.40\dot{7}$ as a fraction in its simplest form.

Let x = 0.4070...
10x = 4.0707...
1000x = 407.0707...
Subtract: 990x = 403
x = $\frac{403}{990}$

**Answer:** $\mathbf{\frac{403}{990}}$

## 5 Conditional probability from a table

The table shows information about 93 employees.
(a) An employee who travels to work by bus is chosen at random. Work out the probability that this employee is a man.
(b) Two of the 93 employees are chosen at random. Work out the probability that both are men.

(a) 28 employees are in the Bus column, and 22 of them are men

Probability = $\frac{11}{14}$

(b) There are 55 men, so P(both) = $\frac{55}{93} \times \frac{54}{92} = \frac{495}{1426}$

**Answer: (a)** $\mathbf{\frac{11}{14}}$ **(b)** $\mathbf{\frac{495}{1426}}$

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## 6 Quadratic sequences

Here are the first five terms of a quadratic sequence.
$-2, -9, -20, -35, -54$
Find an expression for the nth term.

First differences: $-7, -11, -15, -19$
Second differences: $-4, -4, -4$, so $a = -4 \div 2 = -2$
Subtract $-2n^2$ from each term: $0, -1, -2, -3, -4$
The remaining linear sequence has nth term $-n + 1$
nth term = $-2n^2 - n + 1$
**Answer: $-2n^2 - n + 1$**

## 7 Growth and decay

A van is worth £800 when new.
Its value decreases by 8% each year.
After how many whole years will it first be worth less than £550? (Calculator)

Multiplier each year = 0.92
After 4 years: £800 × $0.92^4$ = £573.11
After 5 years: £800 × $0.92^5$ = £527.27
So the first time it is less than £550 is after 5 years
**Answer: 5 years**

## 8 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 2. N is on OB with ON : NB = 1 : 4. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{1}{3}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{1}{3}\mathbf{a} = \frac{1}{3}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{1}{3}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{1}{5}t\mathbf{b}$

Equate: $\frac{1}{3}s = 1 - t$ and $1 - s = \frac{1}{5}t$

Solving: $s = \frac{6}{7}$ and $t = \frac{5}{7}$

OX = $\frac{2}{7}\mathbf{a} + \frac{1}{7}\mathbf{b}$

**Answer: (a) $\frac{1}{3}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{2}{7}\mathbf{a} + \frac{1}{7}\mathbf{b}$**

## 9 Equations in disguise

Solve $x - 10\sqrt{x} + 24 = 0$

Let $u = \sqrt{x}$: $u^2 - 10u + 24 = 0$
Factorise: $(u - 6)(u - 4) = 0$, so $u = 6$ or $u = 4$
$\sqrt{x} = 6$ or $\sqrt{x} = 4$
$x = 36$ or $x = 16$
**Answer: $x = 16$ or $x = 36$**

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## 10 Powers of standard form

Work out $(3.8 \times 10^2)^3$

Give your answer in standard form.

$(3.8 \times 10^2)^3 = 3.8^3 \times (10^2)^3$

$3.8^3 = 54.872$ and $(10^2)^3 = 10^6$

$54.872 \times 10^6 = 5.4872 \times 10^7$

**Answer: $5.4872 \times 10^7$**