Day 208 **S Tier 10-a-day: Model Solutions**

1 Exchange rates

Hassan changes £700 into dollars at £1 = $1.18.
Hassan spends $570 and then changes the rest back into pounds at £1 = $1.25.
How many pounds does Hassan get back?

£700 × 1.18 = $826
826 − 570 = $256
256 ÷ 1.25 = £204.80
Answer: £204.80

2 Recurring decimals

Without using a calculator, work out $0.\dot{8}\dot{5} \times 0.\dot{5}\dot{6}$
Give your answer as a fraction in its simplest form.

$0.\dot{8}\dot{5} = \frac{85}{99}$

$0.\dot{5}\dot{6} = \frac{56}{99}$

$\frac{85}{99} \times \frac{56}{99} = \frac{4760}{9801}$

Answer: $\mathbf{\frac{4760}{9801}}$

3 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = (2x − 15)°, angle ACB = (x + 8)° and angle CAQ = 57°.
(a) Work out the value of x.
(b) Work out the size of angle BAC.
The diagram is not drawn accurately.

(a) Angle ACB = angle PAB (alternate segment theorem)
x + 8 = 2x − 15
x = 23
(b) Angle PAB = 31°
Angles on the straight line PAQ: angle BAC = 180 − 31 − 57 = 92°
Answer: (a) x = 23 (b) 92°

4 Trigonometric graphs

The diagram shows the graph of y = a sin x° + c for 0 ≤ x ≤ 360.
Some points on the graph are labelled.
Find the value of a and the value of c.

At x = 90, sin x° = 1, so a + c = −5
At x = 270, sin x° = −1, so −a + c = −3
Add: 2c = −8, so c = −4
Subtract: 2a = −2, so a = −1
Answer: a = −1, c = −4

5 Averages with algebra

Here are five numbers: 2x − 1, x + 6, 2x + 4, 2x + 6, x − 2.
The mean of the five numbers is 17.
(a) Work out the value of x.
(b) Work out the median of the five numbers.

(a) Total = (2x − 1) + (x + 6) + (2x + 4) + (2x + 6) + (x − 2) = 17 × 5 = 85
8x + 13 = 85, so 8x = 72 and x = 9
The numbers are 17, 15, 22, 24, 7
(b) In order: 7, 15, 17, 22, 24, so the median is 17
Answer: (a) x = 9 (b) 17

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6 Terms of a quadratic sequence

The nth term of a sequence is $n^2 + 4n + 1$.
(a) Work out the 12th term.
(b) Is 399 a term of the sequence? You must show your working.

(a) $12^2 + 4 \times 12 + 1 = 193$

(b) Solve $n^2 + 4n + 1 = 399$, so $n^2 + 4n - 398 = 0$

$b^2 - 4ac = 4^2 - 4 \times 1 \times (-398) = 1608$, which is not a square number

So n is not a whole number, and 399 is not a term

Answer: (a) 193 (b) No

7 Line of best fit

The scatter graph shows distance from town (x) and price (y).
The line of best fit passes through (4, 29) and (6, 28).
(a) Find its equation in the form y = mx + c.
(b) Explain what its gradient tells you.

(a) Gradient $m = \frac{28 - 29}{6 - 4} = \frac{-1}{2} = -0.5$

$y = -0.5x + c$: using (4, 29), $29 = -0.5 \times 4 + c$, so $c = 31$

$y = -0.5x + 31$

(b) For each extra kilometre from the town, the house price (in £10 000s) goes down by 0.5

Answer: (a) $y = -0.5x + 31$ (b) For each extra kilometre from the town, the house price (in £10 000s) goes down by 0.5

8 Rational and irrational numbers

Here are six numbers.

$(1 + \sqrt{2})^2$, $(2 + \sqrt{5})(2 - \sqrt{5})$, $2\pi - \pi$, $\frac{\sqrt{18}}{\sqrt{2}}$, $0.\dot{4}$, $\sqrt{2} + \sqrt{8}$

Write down the rational numbers.

Simplify each number:

$(1 + \sqrt{2})^2 = 3 + 2\sqrt{2}$; $(2 + \sqrt{5})(2 - \sqrt{5}) = 4 - 5 = -1$; $2\pi - \pi = \pi$

$\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{9} = 3$; $0.\dot{4} = \frac{4}{9}$; $\sqrt{2} + \sqrt{8} = 3\sqrt{2}$

Answer: $(2 + \sqrt{5})(2 - \sqrt{5})$, $\frac{\sqrt{18}}{\sqrt{2}}$, $0.\dot{4}$

9 Vectors: intersecting lines

OAB is a triangle. OA = **a** and OB = **b**.
M is on OA with OM : MA = 2 : 3. N is on OB with ON : NB = 1 : 4. AN and BM intersect at X.
(a) Find BM in terms of **a** and **b**.
(b) Find OX in terms of **a** and **b**.
The diagram is not drawn accurately.

(a) $OM = \frac{2}{5}\mathbf{a}$, so $BM = -\mathbf{b} + \frac{2}{5}\mathbf{a} = \frac{2}{5}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so $OX = OB + s\,BM = \frac{2}{5}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: $OX = OA + t\,AN = OA + t(ON - OA) = (1 - t)\mathbf{a} + \frac{1}{5}t\mathbf{b}$

Equate: $\frac{2}{5}s = 1 - t$ and $1 - s = \frac{1}{5}t$

Solving: $s = \frac{20}{23}$ and $t = \frac{15}{23}$

$OX = \frac{8}{23}\mathbf{a} + \frac{3}{23}\mathbf{b}$

Answer: (a) $\frac{2}{5}\mathbf{a} - \mathbf{b}$ (b) $OX = \frac{8}{23}\mathbf{a} + \frac{3}{23}\mathbf{b}$

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10 Equations with algebraic fractions

Solve $\frac{1}{x+2} - \frac{2}{x+5} = \frac{1}{2}$

Multiply every term by $(x + 2)(x + 5)$: $(x + 5) - 2(x + 2) = \frac{1}{2} \times (x + 2)(x + 5)$

Multiply by 2 to clear the fraction: $2(x + 5) - 4(x + 2) = (x + 2)(x + 5)$

Expand and rearrange so that one side is 0: $x^2 + 9x + 8 = 0$

Factorise: $(x + 1)(x + 8) = 0$

$x = -8$ or $x = -1$

Answer: $x = -8$ or $x = -1$