Day 207 **S Tier 10-a-day: Model Solutions**

1 Factorising quadratics

Factorise $12x^2 + 17x - 7$

$12 \times (-7) = -84$. Two numbers that multiply to -84 and add to 17 are 21 and -4

Split the middle term: $12x^2 + 21x - 4x - 7$

Factorise in pairs: $3x(4x + 7) - (4x + 7)$

$= (3x - 1)(4x + 7)$

Answer: $(3x - 1)(4x + 7)$

2 Cyclic quadrilaterals

A, B, C and D are points on a circle. Angle DAB = $(x + 25)°$, angle BCD = $(3x + 47)°$ and angle ABC = $(3x)°$.

(a) Work out the value of x.

(b) Work out the size of angle ADC.

The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°

$(x + 25) + (3x + 47) = 180$

$4x + 72 = 180$, so $4x = 108$

$x = 27$

(b) Angle ABC = $3 \times 27 + 0 = 81°$

Angle ADC = $180 - 81 = 99°$

Answer: (a) x = 27 (b) 99°

3 LCM problems

A church bell rings every 4 minutes.

Another bell rings every 8 minutes.

A third bell rings every 24 minutes.

They all ring together at 6:00 am.

(a) At what time do they next ring together?

Find the LCM of 4, 8 and 24

$4 = 2^2$, $8 = 2^3$, $24 = 2^3 \times 3$

LCM = $2^3 \times 3 = 24$

24 minutes after 6:00 am is 6:24 am

Answer: 6:24 am

4 Simplifying algebraic fractions

Simplify fully $\frac{5x^2 - 180}{x^2 - x - 30}$

Factorise the top: $5(x - 6)(x + 6)$

Factorise the bottom: $(x - 6)(x + 5)$

Cancel the common factor $(x - 6)$: $\frac{5(x + 6)}{x + 5}$

Answer: $\frac{5(x + 6)}{x + 5}$

5 Pressure

A stone block in the shape of a cuboid weighs 690 N.

Its smallest face has area 400 cm^2. Its largest face has area 1,500 cm^2.

Pressure = force ÷ area

Work out (a) the greatest pressure and (b) the least pressure the stone block can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 400 cm^2 = 0.04 m^2 and 1,500 cm^2 = 0.15 m^2

(a) Greatest pressure is on the smallest face: $690 \div 0.04 = 17{,}250$ N/m^2

(b) Least pressure is on the largest face: $690 \div 0.15 = 4{,}600$ N/m^2

Answer: (a) 17,250 N/m^2 (b) 4,600 N/m^2

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6 Profit and loss

A trader buys a watch and adds a 25% mark-up to get the selling price.
In a sale the selling price is reduced by 30% to £131.25.
Work out how much the shop paid for a watch.

Overall multiplier = 1.25 × 0.7 = 0.875
Cost = £131.25 ÷ 0.875
= £150
Answer: £150

7 Exponential growth and decay

A radioactive sample has a mass of 480 g. Its mass halves every 3 hours.
(a) Work out the mass after 12 hours.
(b) After how many hours will the mass first be less than 25 g?

(a) 12 hours is 4 half-lives: $480 \div 2^4 = 30$ g
(b) Halve repeatedly: 480, 240, 120, 60, 30, 15
15 is the first value below 25, after 5 half-lives, so 5 × 3 = 15 hours
Answer: (a) 30 g (b) 15 hours

8 Perpendicular bisector

A is the point $(-1, -2)$ and B is the point $(-7, -4)$.
Find an equation of the perpendicular bisector of AB. Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = $(-4, -3)$
Gradient of AB = $\frac{1}{3}$
Perpendicular gradient = -3
$y - (-3) = (-3)(x - (-4))$
Rearrange to integer coefficients: $3x + y = -15$
Answer: $3x + y = -15$

9 Surds with unknowns

Given that $(a + \sqrt{14})(b + \sqrt{14}) = 26 + 8\sqrt{14}$,
where a and b are positive integers with a < b, find the values of a and b.

$(a + \sqrt{14})(b + \sqrt{14}) = ab + (a + b)\sqrt{14} + 14$
Compare the $\sqrt{14}$ terms: $a + b = 8$
Compare the integer terms: $ab + 14 = 26$, so $ab = 12$
Two numbers with sum 8 and product 12 are 2 and 6
Answer: a = 2, b = 6

10 Loci and regions

ABCD is a rectangle with AB = 9 cm and AD = 9 cm.
A point P inside the rectangle is such that AP ≤ 8 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 8 cm: P is inside the quarter circle of radius 8 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves
Area = $\frac{1}{2} \times \frac{1}{4} \times \pi \times 8^2 = 8\pi$ cm^2
Answer: 8π cm^2