Day 205 **S Tier 10-a-day: Model Solutions**

1 Sine rule

The diagram shows triangle ABC. (Calculator)
Angle BAC = 46° and angle ABC is acute.
Work out the size of angle ABC, marked θ. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{\sin\theta}{AC} = \frac{\sin A}{BC}$

$\frac{\sin\theta}{5} = \frac{\sin 46°}{9}$

$\sin\theta = \frac{5 \times \sin 46°}{9} = 0.39963$

$\theta = \sin^{-1}(0.39963) = 23.6°$ (1 d.p.)

Answer: $\theta = 23.6°$

2 Iteration: rearranging equations

(a) Show that the equation $x^2 - 7x - 7 = 0$ can be rearranged to give $x = 7 + \frac{7}{x}$ (Calculator)

(b) Use the iteration formula $x_{n+1} = 7 + \frac{7}{x_n}$ with $x_0 = 7$ to work out x_1 and x_2. Give each answer correct to 3 decimal places.

(a) Rearrange $x^2 - 7x - 7 = 0$: $x^2 - 7x = 7$

Factorise: $x(x - 7) = 7$

Divide by x: $x - 7 = \frac{7}{x}$, so $x = 7 + \frac{7}{x}$

(b) $x_1 = 7 + \frac{7}{7} = 8.00000...$

$x_2 = 7 + \frac{7}{x_1} = 7.87500...$

$x_1 = 8.000$ and $x_2 = 7.875$ (3 d.p.)

Answer: (b) $x_1 = 8.000$, $x_2 = 7.875$

3 Rationalising the denominator

Rationalise the denominator of $\frac{27}{3\sqrt{2}}$

Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{2}$:

$\frac{27}{3\sqrt{2}} = \frac{27\sqrt{2}}{6}$

$= \frac{9\sqrt{2}}{2}$

Answer: $\frac{9\sqrt{2}}{2}$

4 Fractions removed from a ratio

The ratio of red to blue marbles in a jar is 7 : 8.

Then $\frac{2}{3}$ of the red and $\frac{3}{4}$ of the blue are taken out.

Work out the ratio of red to blue now.

Left: $\frac{1}{3} \times 7 = \frac{7}{3}$ of the red part, $\frac{1}{4} \times 8 = 2$ of the blue part

$\frac{7}{3} : 2$

Multiply by 3: 7 : 6

Answer: 7 : 6

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5 Pythagoras with algebra

ABC is a right-angled triangle with sides (x + 1) cm, (2x) cm and (2x + 1) cm. The longest side is (2x + 1) cm.
(a) Find the value of x.
(b) Work out the area of the triangle.
The diagram is not drawn accurately.

(a) Pythagoras: $(x + 1)^2 + (2x)^2 = (2x + 1)^2$
Expand and collect terms: $x^2 - 2x = 0$
Factorise: $x(x - 2) = 0$
x = 2 or x = 0, and x must be positive, so x = 2
(b) The two shorter sides are 3 cm and 4 cm
Area = $\frac{1}{2} \times 3 \times 4 = 6$ cm^2

Answer: (a) x = 2 (b) 6 cm^2

6 Reverse percentages

The price of a coat is increased by 40%.
Later the new price is reduced by 40% in a sale.
The sale price is £218.40.
Work out the price before the increase. (Calculator)

Multipliers: 1.4 and 0.6
Overall multiplier = 1.4 × 0.6 = 0.84
Original × 0.84 = £218.40
Original = £218.40 ÷ 0.84 = £260
Answer: £260

7 Functions that are their own inverse

$f(x) = \frac{x + 7}{2x - 1}$ where $x \neq \frac{1}{2}$
(a) Show that ff(x) = x.
(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by (2x − 1)
$ff(x) = \frac{1(x + 7) + 7(2x - 1)}{2(x + 7) - 1(2x - 1)}$
Top = x + 7 + 14x − 7 = 15x
Bottom = 2x + 14 − 2x + 1 = 15
$ff(x) = \frac{15x}{15} = x$

(b) ff(x) = x, so f is its own inverse: $f^{-1}(x) = \frac{x + 7}{2x - 1}$

Answer: (b) $f^{-1}(x) = \frac{x + 7}{2x - 1}$

8 Percentages of percentages

In a group of members, 60% are juniors and the rest are seniors.
25% of the juniors and 60% of the seniors play chess.
What percentage of all the members play chess?

Take 100 members: 60 are juniors and 40 are seniors
25% of 60 = 15 and 60% of 40 = 24
15 + 24 = 39 out of 100
Answer: 39%

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9 Estimated mean with an unknown frequency

The table gives information about the masses of some eggs.
An estimate for the mean is 53.
Work out the value of x.

Mid-points: 20, 40, 60, 80
Estimated mean = $\Sigma fm \div \Sigma f$, so $(1040 + 60x) \div (22 + x) = 53$
$1040 + 60x = 53(22 + x) = 1166 + 53x$
$(60 - 53)x = 1166 - 1040$
$7x = 126$, so $x = 18$
Answer: x = 18

10 Finding a percentage rate

A car was bought for £800.
Each year its value decreases by the same percentage.
After 2 years it is worth £288.
Work out the percentage decrease each year.

Multiplier for 2 years = £288 ÷ £800 = 0.36
Yearly multiplier = the square root of 0.36 = 0.6
40% decrease each year
Answer: 40%