Day 204 **S Tier 10-a-day: Model Solutions**

1 Mean of a changing set

The mean of 9 numbers is 52.
A new number is added and the mean of all 10 numbers is 47.8.
Work out the new number.

Total of the 9 numbers = 9 × 52 = 468
New total = 10 × 47.8 = 478
New number = 478 − 468 = 10
Answer: 10

2 Solving quadratics by factorising

Solve $12x^2 + 28 = 37x$

Rearrange so that one side is 0: $12x^2 - 37x + 28 = 0$
Factorise: $(3x - 4)(4x - 7) = 0$
$3x - 4 = 0$ or $4x - 7 = 0$
$x = \frac{4}{3}$ or $x = \frac{7}{4}$
Answer: $x = \frac{4}{3}$ or $x = \frac{7}{4}$

3 Cyclic quadrilaterals

A, B, C and D are points on a circle. Angle DAB = (x + 30)°, angle BCD = (3x + 38)° and angle ABC = (3x − 25)°.
(a) Work out the value of x.
(b) Work out the size of angle ADC.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
(x + 30) + (3x + 38) = 180
4x + 68 = 180, so 4x = 112
x = 28
(b) Angle ABC = 3 × 28 − 25 = 59°
Angle ADC = 180 − 59 = 121°
Answer: (a) x = 28 (b) 121°

4 Simultaneous equations

Solve the simultaneous equations
x + 4y = 27
−5x + 6y = 21

To match the x terms: (1) × 5 gives 5x + 20y = 135
Add the equations to eliminate x: 26y = 156
y = 6
Substitute y = 6 into (1): x + 24 = 27
x = 3
Answer: x = 3, y = 6

5 Relative speed

Hassan sets off at 9:00 and cycles at 8 km/h along a path.
Amir sets off from the same place at 9:45 and cycles at 16 km/h along the same path.
At what time does Amir catch up with Hassan?

When Amir starts, Hassan is 0.75 × 8 = 6 km ahead
Amir gains 16 − 8 = 8 km every hour
Time to catch up = 6 ÷ 8 = 0.75 hours = 45 minutes
9:45 + 45 minutes = 10:30
Answer: 10:30

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6 Reverse percentages

The number of members of a club is decreased by 20% in one year.
The next year the number of members is decreased by 25%.
There are now 720 members.
How many members were there to begin with?

Multipliers: 0.8, 0.75
Overall multiplier = 0.8 × 0.75 = 0.6
Original = final ÷ multiplier = 720 ÷ 0.6
= 1200
Answer: 1200

7 Transformations of graphs

The graph of y = f(x) has a minimum point at (1, −3).
Write down the coordinates of the turning point of the graph of
(a) y = f(x − 3)
(b) y = f(2x) − 2

(a) Start with the turning point (1, −3)
Apply the transformation to the point: (4, −3)

(b) f(2x) changes the x-coordinate to $\frac{1}{2}$, and f(x) − 2 changes the y-coordinate to −5

Turning point: $(\frac{1}{2}, -5)$

Answer: (a) (4, −3) (b) $(\frac{1}{2}, -5)$

8 Enlargement

Triangle P has vertices A(2, 2), B(3, 0) and C(2, 3).
Triangle Q has vertices A'(−2, 10), B'(−5, 16) and C'(−2, 7).
Q is an enlargement of P.
Describe fully the single transformation that maps P onto Q.

x-step from A to B = 1, x-step from A' to B' = −3
Scale factor = −3 ÷ 1 = −3
The centre C satisfies: C + −3(A − C) = A', so (1 − −3)C = A' − −3 A
x: 4 × x = (−2) − (−3) × 2 = 4, so x = 1
y: 4 × y = 10 − (−3) × 2 = 16, so y = 4
Answer: Enlargement, scale factor −3, centre (1, 4)

9 Tangent to a circle: proof

Show that the line y = 2x + 15 is a tangent to the circle $x^2 + y^2 = 45$.
Find the coordinates of the point where the line touches the circle.

Substitute y = 2x + 15 into $x^2 + y^2 = 45$: $x^2 + (2x + 15)^2 = 45$
Expand: $5x^2 + 60x + 180 = 0$
$b^2 - 4ac = 60^2 - 4 \times 5 \times 180 = 3600 - 3600 = 0$
$b^2 - 4ac = 0$, so there is exactly one point of intersection: the line is a tangent
$x = \frac{-60}{2 \times 5} = -6$, and y = 2 × (−6) + 15 = 3. The point is (−6, 3)
Answer: (−6, 3)

10 Nested surds

Show that $\sqrt{79 + 30\sqrt{6}} = 5 + 3\sqrt{6}$

Square the right-hand side: $(5 + 3\sqrt{6})^2 = 25 + 30\sqrt{6} + 54$
$= 79 + 30\sqrt{6}$
So $\sqrt{79 + 30\sqrt{6}} = 5 + 3\sqrt{6}$
Answer: $\sqrt{79 + 30\sqrt{6}} = 5 + 3\sqrt{6}$