Day 203 **S Tier 10-a-day: Model Solutions**

1 Probability and quadratic equations

A bag contains some counters. 6 of them are green and the rest are white.
Two counters are taken at random without replacement.

The probability that both are green is $\frac{15}{28}$.

Work out how many counters are white.

Let there be n counters altogether

P(both green) = $\frac{6}{n} \times \frac{5}{n-1} = \frac{30}{n(n-1)}$

$\frac{30}{n(n-1)} = \frac{15}{28}$, so $n(n-1) = 56$, giving $n^2 - n - 56 = 0$

$(n-8)(n+7) = 0$, so $n = 8$ (n cannot be negative)

Number of counters that are white = $8 - 6 = 2$

Answer: 2

2 Angles with algebra

ABC is an isosceles triangle with AB = AC.
Angle BAC = $(5x + 10)°$
Angle ABC = $(2x + 22)°$
(a) Work out the value of x.
(b) Work out the size of angle BAC.

(a) Angles ABC and ACB are equal, and the angles in a triangle add up to 180°

$(5x + 10) + 2(2x + 22) = 180$

$9x + 54 = 180$, so $9x = 126$

$x = 14$

(b) Angle BAC = $5 \times 14 + 10 = 80°$

Answer: (a) x = 14 (b) 80°

3 LCM problems

A red light flashes every 6 minutes.
A blue light flashes every 8 minutes.
A green light flashes every 18 minutes.
They all flash together at 6:00 am.
How many times do they all flash together between 6:00 am and 3:00 pm, including both 6:00 am and 3:00 pm if they do?

Find the LCM of 6, 8 and 18

$6 = 2 \times 3$, $8 = 2^3$, $18 = 2 \times 3^2$

LCM = $2^3 \times 3^2 = 72$

Total time from 6:00 am to 3:00 pm = 540 minutes

$540 \div 72 = 7.5$, so there are 7 more times after the start

Including the start: 7 + 1 = 8

Answer: 8

4 Sine rule

The diagram shows triangle ABC. (Calculator)
Work out the length of AC. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{AC}{\sin B} = \frac{BC}{\sin A}$

$\frac{x}{\sin 53°} = \frac{8}{\sin 46°}$

$x = \frac{8 \times \sin 53°}{\sin 46°} = 8.8819$

x = 8.9 cm (1 d.p.)

Answer: AC = 8.9 cm

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5 Powers of standard form

Work out $(2.7 \times 10^{-2})^3$
Give your answer in standard form.

$(2.7 \times 10^{-2})^3 = 2.7^3 \times (10^{-2})^3$
$2.7^3 = 19.683$ and $(10^{-2})^3 = 10^{-6}$
$19.683 \times 10^{-6} = 1.9683 \times 10^{-5}$
Answer: 1.9683×10^{-5}

6 Rearranging when the subject appears twice

Make x the subject of $y = \frac{6x - 6}{3x - 4}$

Multiply by $3x - 4$: $y(3x - 4) = 6x - 6$
Expand: $3xy - 4y = 6x - 6$
Collect the x terms: $3xy - 6x = -6 + 4y$
Factorise: $x(3y - 6) = 4y - 6$
$x = \frac{4y - 6}{3y - 6}$
Answer: $x = \frac{4y - 6}{3y - 6}$

7 Ratio and expressions

x : y = 8 : 5
Work out the ratio (6x − 3y) : (2x + 1y)
Give your answer in its simplest form.

x : y = 8 : 5, so let x = 8t and y = 5t
6(8t) − 3(5t) = 33t
2(8t) + 1(5t) = 21t
33t : 21t = 11 : 7
Answer: 11 : 7

8 Bounds of area and perimeter

A rectangle has length 25 cm and width 7 cm.
Both measurements are correct to the nearest centimetre.
Kofi says that the perimeter of the rectangle could be exactly 64 cm.
Is Kofi correct? You must show your working.

$24.5 \leq$ length < 25.5 and $6.5 \leq$ width < 7.5
Lower bound of perimeter = $2 \times 24.5 + 2 \times 6.5 = 62$
Upper bound of perimeter = $2 \times 25.5 + 2 \times 7.5 = 66$
So $62 \leq$ perimeter < 66
64 is in this interval, so Kofi is correct
Answer: Yes, 64 is between 62 and 66

9 Equal roots and real roots

The equation $2x^2 + 6x + k = 0$ has two different real solutions.
Find the range of possible values of k.

Two different real solutions means $b^2 - 4ac > 0$
$6^2 - 4 \times 2 \times k > 0$, so $36 - 8k > 0$
$36 > 8k$, so $k < \frac{9}{2}$
Answer: $k < \frac{9}{2}$

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10 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 1. N is on OB with ON : NB = 3 : 1. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{1}{2}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{1}{2}\mathbf{a} = \frac{1}{2}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{1}{2}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{3}{4}t\mathbf{b}$

Equate: $\frac{1}{2}s = 1 - t$ and $1 - s = \frac{3}{4}t$

Solving: $s = \frac{2}{5}$ and $t = \frac{4}{5}$

OX = $\frac{1}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$

Answer: (a) $\frac{1}{2}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{1}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$