Day 202 **S Tier 10-a-day: Model Solutions**

1 Cone made from a sector

A sector of a circle has radius 12 cm and angle 90°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{90}{360} \times 2\pi \times 12 = 6\pi$ cm

This is the circumference of the base: $2\pi r = 6\pi$, so r = 3 cm

(b) Slant height = 12 cm, so height$^2 = 12^2 - 3^2 = 135$, height = $3\sqrt{15}$ cm

Volume = $\frac{1}{3} \times \pi \times 3^2 \times 3\sqrt{15} = 9\sqrt{15}\pi$ cm^3

Answer: (a) 3 cm (b) $9\sqrt{15}\pi$ cm^3

2 Equations with fractions

Solve $\frac{x+2}{2} + \frac{x-4}{4} = 6$

Multiply every term by 4: $2(x + 2) + (x - 4) = 24$
Expand: $3x = 24$
$3x = 24$
$x = 8$
Answer: x = 8

3 Time zones

A flight leaves London at 17:50 (London time).
It takes 12 hours 10 minutes.
When it is 12:00 in London it is 07:00 in New York.
At what time, in New York time, does the flight land? Say if it is on a different day.

Arrival in London time = 17:50 + 12 h 10 min = 06:00 the next day
New York is 5 hours behind London, so subtract 5 hours
01:00 the next day
Answer: 01:00 the next day

4 Changing the subject

Make u the subject of $v = \frac{6}{\sqrt{u}}$

Multiply by $\sqrt{u}$: $v\sqrt{u} = 6$

Divide by v: $\sqrt{u} = \frac{6}{v}$

Square both sides: $u = \frac{36}{v^2}$

Answer: $u = \frac{36}{v^2}$

5 Pressure

A concrete slab in the shape of a cuboid weighs 870 N.
Its smallest face has area 200 cm^2. Its largest face has area 1,500 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the concrete slab can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 200 cm^2 = 0.02 m^2 and 1,500 cm^2 = 0.15 m^2

(a) Greatest pressure is on the smallest face: 870 ÷ 0.02 = 43,500 N/m^2

(b) Least pressure is on the largest face: 870 ÷ 0.15 = 5,800 N/m^2

Answer: (a) 43,500 N/m^2 (b) 5,800 N/m^2

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6 Exact lengths and areas

A rectangle has area $(34 + 14\sqrt{5})$ cm^2 and length $(4 + 2\sqrt{5})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{5}$

Width = area ÷ length = $\frac{34 + 14\sqrt{5}}{4 + 2\sqrt{5}}$

Multiply top and bottom by $(4 - 2\sqrt{5})$:
Numerator: $(34 + 14\sqrt{5})(4 - 2\sqrt{5}) = -4 - 12\sqrt{5}$
Denominator: $16 - 20 = -4$
Width = $1 + 3\sqrt{5}$
Answer: $1 + 3\sqrt{5}$ cm

7 Area of a segment

The circle has centre O and radius 11 cm. A and B are points on the circle and angle AOB = 130°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{130}{360} \times \pi \times 11^2 = 137.27$ cm^2

Area of triangle OAB = $\frac{1}{2} \times 11 \times 11 \times \sin 130° = 46.346$ cm^2

Segment = sector − triangle = 137.27 − 46.346 = 90.924
= 90.9 cm^2 (3 s.f.)
Answer: 90.9 cm^2

8 At least one

The probability that a bus is more than 5 minutes late on any morning is $\frac{3}{20}$.

Events on different occasions are independent.
Work out the probability that this happens at least once in 4 mornings.
Give your answer correct to 3 significant figures. (Calculator)

P(never) = $(1 - \frac{3}{20})^4 = \frac{17}{20}^4 = 0.52200625$
P(at least once) = 1 − P(never) = 1 − 0.52200625
= 0.478
Answer: 0.478

9 Surds and fractions

Simplify $\frac{1}{3 + \sqrt{6}} - \frac{1}{3 - \sqrt{6}}$
Give your answer with a rational denominator.

Common denominator: $(3 + \sqrt{6})(3 - \sqrt{6}) = 9 - 6 = 3$
Numerator: $(3 - \sqrt{6}) - (3 + \sqrt{6})$
$= -2\sqrt{6}$
$\frac{-2\sqrt{6}}{3} = -\frac{2\sqrt{6}}{3}$
Answer: $-\frac{2\sqrt{6}}{3}$

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10 Tangency conditions

The line $y = 2x + k$ is a tangent to the circle $x^2 + y^2 = 12$.
Find the possible values of k.
Give any surds in their simplest form.

Substitute $y = 2x + k$: $x^2 + (2x + k)^2 = 12$

Expand: $5x^2 + 4kx + k^2 - 12 = 0$

A tangent touches at one point, so $b^2 - 4ac = 0$: $(4k)^2 - 4 \times 5 \times (k^2 - 12) = 0$

$-4k^2 + 240 = 0$, so $k^2 = 60$

$k = \pm 2\sqrt{15}$

Answer: $k = \pm 2\sqrt{15}$