Day 201 **S Tier 10-a-day: Model Solutions**

1 Ordering numbers

Write these numbers in order of size, starting with the smallest.

$\frac{9}{17}$, 0.65, $\frac{8}{13}$, $\frac{5}{12}$

Convert each to a decimal: $\frac{9}{17} = 0.529$, $0.65 = 0.65$, $\frac{8}{13} = 0.615$, $\frac{5}{12} = 0.417$

In order: $\frac{5}{12}$, $\frac{9}{17}$, $\frac{8}{13}$, 0.65

Answer: $\frac{5}{12}$, $\frac{9}{17}$, $\frac{8}{13}$, 0.65

2 Capture and recapture

80 deer in a forest are caught, marked and put back.
Later, a second sample of 30 deer is caught. 5 of these are marked.
(a) Work out an estimate for the number of deer in a forest.
(b) State one assumption you have made.

(a) The proportion marked in the second sample is 5 out of 30

This should match the proportion marked in the whole population: $\frac{5}{30} = \frac{80}{N}$

$N = 80 \times 30 \div 5 = 480$

(b) No deer are born, die or leave between the samples, and the deer mix evenly

Answer: (a) 480 (b) The population stays the same between samples

3 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 64°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 64°
Angle COB is an exterior angle of triangle OAC: 64 + 64 = 128°
(or: the angle at the centre is twice the angle at the circumference)

Answer: (a) 90° (b) 128°

4 Inequalities

(a) Solve $7 - 6x < -17$
(b) n is an integer. Write down all the values of n that satisfy
$3 \leq 3n - 3 < 12$

(a) Subtract 7: $-6x < -24$
Divide by -6 and reverse the inequality sign: $x > 4$
(b) Add 3: $6 \leq 3n < 15$
Divide by 3: $2 \leq n < 5$
Integers: 2, 3, 4

Answer: (a) $x > 4$ (b) 2, 3, 4

5 HCF and LCM

(a) Write 432 and 1323 as products of prime factors.
(b) Find the HCF and the LCM of 432 and 1323.
Give your answers as numbers.

(a) $432 = 2^4 \times 3^3$ and $1323 = 3^3 \times 7^2$

(b) HCF = $3^3 = 27$

LCM = $2^4 \times 3^3 \times 7^2 = 21{,}168$

Answer: (a) $2^4 \times 3^3$; $3^3 \times 7^2$ (b) HCF = 27, LCM = 21,168

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6 Algebraic proof

Prove algebraically that $(3n + 1)^2 - 1$ is a multiple of 3 for all integer values of n.

$(3n + 1)^2 - 1 = 9n^2 + 6n + 1 - 1$

$= 9n^2 + 6n$

$= 3(3n^2 + 2n)$, which is a multiple of 3

Answer: Proved

7 Rationalising the denominator

Write $\frac{15}{3 + \sqrt{3}}$ in the form $\frac{p + q\sqrt{3}}{r}$

where p, q and r are integers and r > 1,
and the fraction is in its simplest form.

Multiply top and bottom by $(3 - \sqrt{3})$:

$\frac{15}{3 + \sqrt{3}} = \frac{15(3 - \sqrt{3})}{(3 + \sqrt{3})(3 - \sqrt{3})}$

Denominator: $9 - 3 = 6$

$= \frac{15 - 5\sqrt{3}}{2}$

Answer: $\mathbf{\frac{15 - 5\sqrt{3}}{2}}$

8 Comparing costs

Hire Shop A has a delivery charge of £7 plus £1.50 per day.
Hire Shop B has a delivery charge of £13.50 plus £1 per day.
(a) Work out the number of days for which the two costs are the same.
(b) Which is cheaper for 21 days, and by how much?

(a) Let n be the number of days. Working in pounds, the costs are equal when $7 + 1.50n = 13.50 + n$
$0.50n = 6.50$, so $n = 13$
(b) Hire Shop A: £7 + £1.50 × 21 = £38.50
Hire Shop B: £13.50 + £1 × 21 = £34.50
Hire Shop B is cheaper by £4
Answer: (a) 13 (b) Hire Shop B, by £4

9 Tangent to a circle

A circle has equation $x^2 + y^2 = 841$.
The point P(−20, −21) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $\frac{21}{20}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div \frac{21}{20} = -\frac{20}{21}$

$y - (-21) = (-\frac{20}{21})(x - (-20))$

Multiply out and rearrange: $20x + 21y = -841$
(b) On the x-axis y = 0, so $20x = -841$
$x = -\frac{841}{20}$, so T is $(-\frac{841}{20}, 0)$

Answer: (a) $\mathbf{20x + 21y = -841}$ **(b) T =** $\mathbf{(-\frac{841}{20}, 0)}$

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10 Probability from a frequency tree

In a population of 10,000 people, 4% have a certain disease.
A test gives a positive result for 90% of people who have the disease and for 4% of people who do not.
A person is chosen at random and tests positive.
Work out the probability that they have the disease.

4% of 10,000 = 400 have the disease and 9,600 do not
Flagged and genuine: 400 × 90% = 360
Flagged but not: 9,600 × 4% = 384
Total flagged or positive = 360 + 384 = 744

Probability = 360 ÷ 744 = $\frac{15}{31}$

Answer: $\mathbf{\frac{15}{31}}$